Collective flow in a hot, dense and strongly interacting medium

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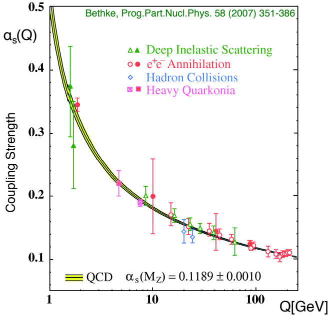
High temperature → deconfined quarks and gluons

Quantum-chromo-dynamics (QCD) tells us that quarks and gluons are asymptotically free at high energies



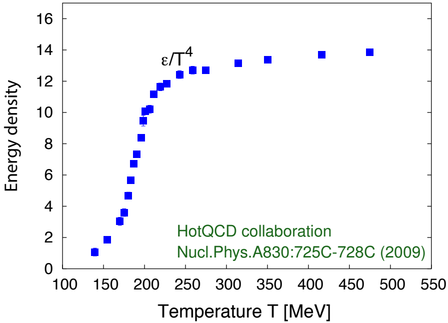
Energy Scale

Small energy
long distance
Coupling large

→ Confinement

High energy
short distance
Coupling small
→ Asymptotic Freedom

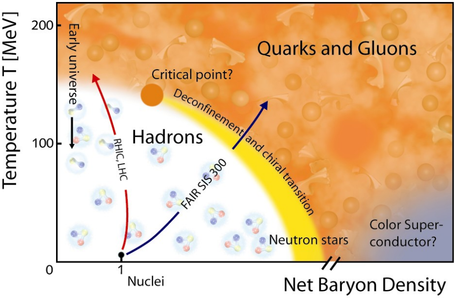
Lattice calculation - rapid crossover



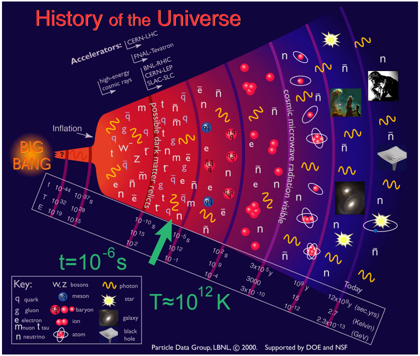
Extreme matter in heavy-ion collisions

Heavy-ion collisions (HIC) generate extended systems
at high enough temperatures to create the quark-gluon plasma

Phase diagram of strongly interacting matter

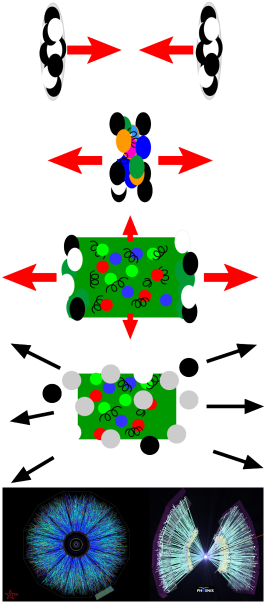


GSI: www.gsi.de



HICs allow us to study complex many-body systems governed by QCD
and understand fundamental properties of matter

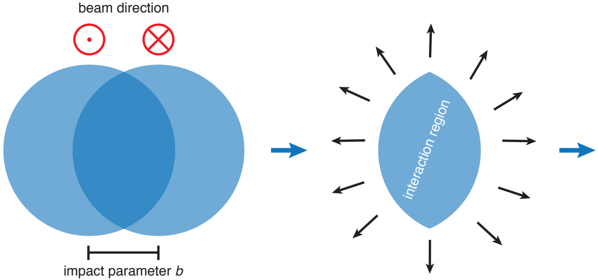
Timeline of a Heavy-Ion Collision



	before collision	**Initial state** nuclei at high energy (color glass condensate)
0 fm/c		
	pre-equilibrium	Thermalization? (glasma state)
~ 0.5 fm/c		
	quark-gluon-plasma	**Hydrodynamics** Jet quenching ...
$\sim 3-5$ fm/c		
	hadronization hadr.rescattering	Hadronization models Hadronic transport
~ 10 fm/c		
	freeze-out	
	detection	**Compare theory to experiment**

Interesting observable: Azimuthal particle distribution

... angular distribution of particles perpendicular to the beam direction



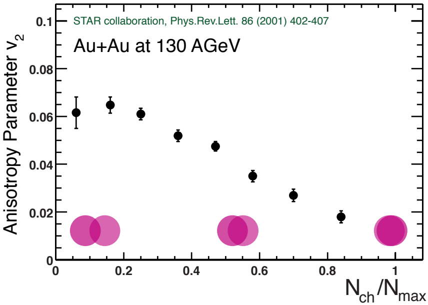
measured $\frac{dN}{d\phi}$

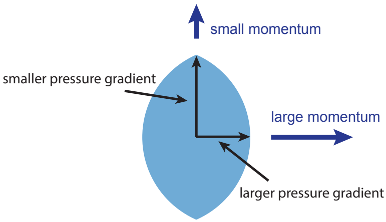
Quantify anisotropy using Fourier decomposition of the distribution:

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n (2v_n \cos(n\phi))\right) \Rightarrow v_2 \text{ characterizes elliptic shape}$$

Large azimuthal anisotropy: system flows

Result: large anisoptropy, depends on initial geometry





Data is explained if system behaves like an *almost ideal fluid*

System is strongly interacting

Weakly interacting system would have very small anisotropy

Relativistic fluid-dynamics

So, the bulk of the matter produced in heavy-ion collisions is well described by fluid-dynamics...

... and that is just energy and momentum conservation
in a system with small mean free path (compared to the system size)

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}$$

need additional equation to close the set:

Equation of state: $P = P(\epsilon)$

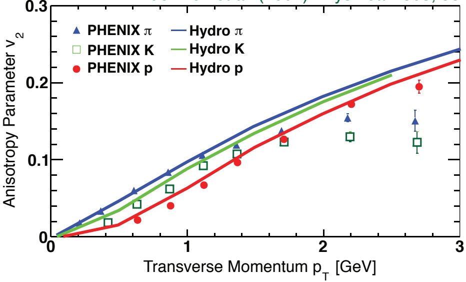
from lattice QCD / hadron gas model

$\pi^{\mu\nu}$ contains dissipative effects

Short history of flow: 1992-2004

- Positive v_2 predicted for high energy collisions
  J.-Y. Ollitrault, Phys.Rev. D46 (1992) 229-245
- First observed at AGS, then SPS
  E877, J. Barrette et al., PRL 73, 2532 (1994), NA49, T. Wienold et al., Nucl. Phys. A610, 76c (1996)
- Ideal hydrodynamics ($\pi^{\mu\nu} = 0$) has been very successful in describing flow and other observables at RHIC
  Kolb and Heinz, Quark Gluon Plasma 3 (Singapore: World Scientific), 634 (2003)
  Huovinen, Quark Gluon Plasma 3 (Singapore: World Scientific), 600 (2003)

Huovinen et al. (2001) Phys.Lett.B503, 58



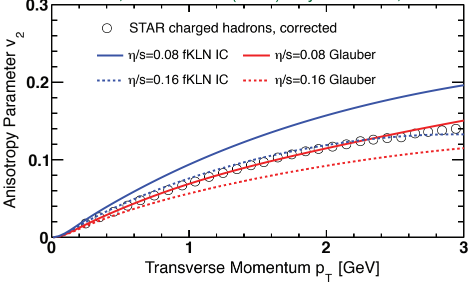
Shows that system is strongly interacting

Calculation was missing important ingredients: QCD equation of state, viscosity, fluctuations, ...

Short history of flow: 2005-2008

- Viscosity can not be zero. AdS/CFT correspondence provides "lower bound" of the shear viscosity to entropy density ratio $\eta/s = 1/4\pi$ in N=4 SYM theory Kovtun, Son, Starinets, Phys.Rev.Lett. 94 (2005) 111601
- Early estimates for heavy-ion collisions give $\eta/s = 0.08 - 0.34$
Teaney, Phys.Rev. C68, 034913 (2003), Gavin and Abdel-Aziz, Phys.Rev.Lett. 97, 162302 (2006)
Lacey and Taranenko, PoS CFRNC2006, 021 (2006), Gelman, Shuryak and Zahed, Phys.Rev. C74, 044908 (2006) ...
- First relativistic viscous hydrodynamic simulation shows: η/s has to be small indeed P. Romatschke and U. Romatschke, Phys.Rev.Lett. 99 (2007) 172301

Luzum, Romatschke (2008) Phys.Rev.C78, 034915



Using smooth initial energy density distributions with different geometries

Extracted η/s depends on initial state model

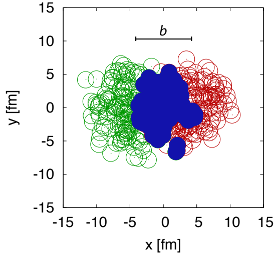
Short history of flow: 2008-2010

- Fluctuations are realized to be important
  They affect all harmonics v_1, v_2, v_3, ...

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n 2v_n \cos(n(\phi - \psi_n))\right)$$

  In particular: odd harmonics are not zero
  Mishra et al., Phys.Rev. C77, 064902 (2008), Takahashi et al., Phys. Rev. Lett. 103, 242301 (2009)
  Alver and Roland, Phys. Rev. C81, 054905 (2010)

- Axes ψ_n and eccentricities determined by fluctuating geometry



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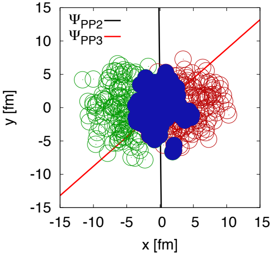

Eccentricities:

$$\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$$

All $\varepsilon_n = 0 \to$ perfect circle
$\varepsilon_n \neq 0$: shape modulation in the $n^{\rm th}$ harmonic

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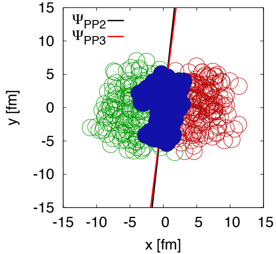

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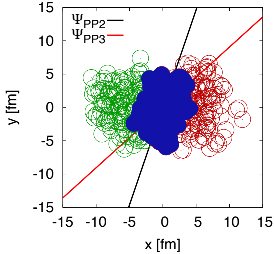

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3+1D event-by-event viscous fluid-dynamics

The first relativistic hydrodynamic simulation to include viscosity, fluctuations and 3+1 dimensions:

MUSIC: MUScl for Ion Collisions

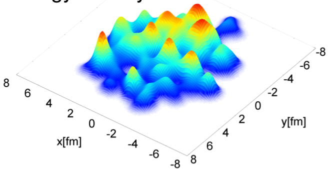
MUSCL = Monotonic Upstream Centered Scheme for Conservation Laws

B. Schenke, S. Jeon, and C. Gale, Phys.Rev.Lett.106, 042301 (2011)

- 3+1 dimensions
- expanding geometry
- viscous effects (2^{nd} order Israel-Stewart formalism)
- fluctuating initial conditions
- equation of state from lattice QCD and hadron resonance gas

Studying the effect of viscosity and fluctuations

- Setup: simple initial state
  Wounded nucleons are assigned
  a Gaussian energy density distribution
  width σ_0 is a free parameter



- Evolution:
  Hydrodynamic evolution with shear viscous effects
  System expands, becomes dilute, freezes out
  Initial spatial anisotropy is transformed into momentum anisotropy
- Energy density $\rightarrow$ Particle spectra:
  Cooper-Frye formula: Cooper and Frye, Phys.Rev.D10, 186 (1974)

$$E\frac{dN}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(T, p_\mu u^\mu, \pi^{\mu\nu})$$

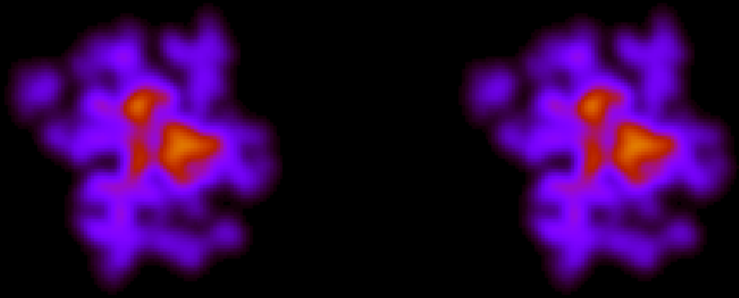
$\Sigma =$ freeze-out surface (surface of constant temperature)
$f =$ particle distribution

+ resonance decays

Evolution: Effect of Viscosity

B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

ideal $\qquad\qquad$ $\eta/s = 0.16$

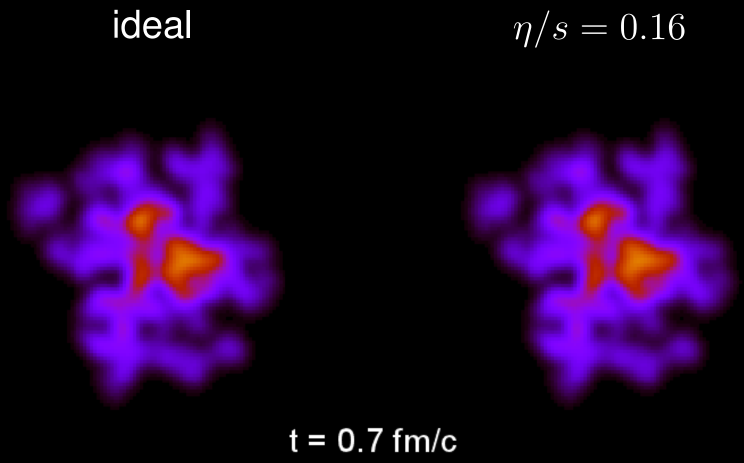


t = 0.7 fm/c

energy density (scale adjusted with time)

Evolution: Effect of Viscosity

B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

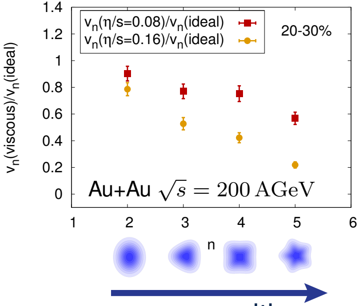


energy density (scale adjusted with time)

Sensitivity of v_n on viscosity and fluctuations

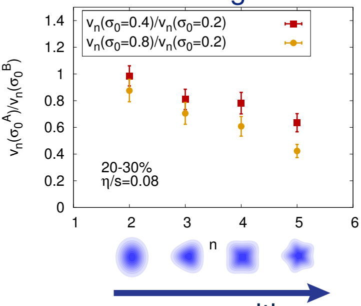
B. Schenke, S. Jeon, C. Gale, Phys.Rev.C85, 024901 (2012)

viscous to ideal results



more sensitive

smoother to more granular results

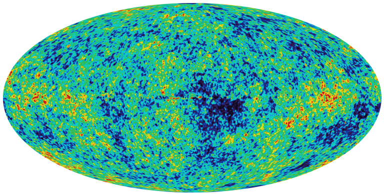


more sensitive

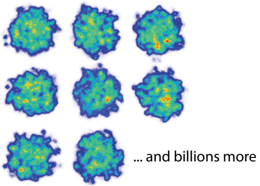
Viscosity decreases anisotropic flow (it's friction)
Smoother initial conditions decrease anisotropic flow

Sensitivity to viscosity and initial state structure increases with n

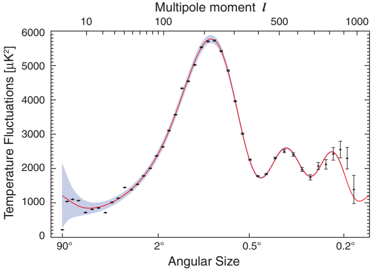
Analogy: Cosmic microwave background

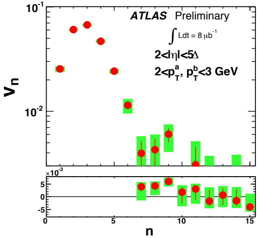


CMB Credit: WMAP Science Team, NASA



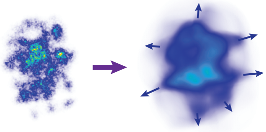
Heavy-Ion Collision



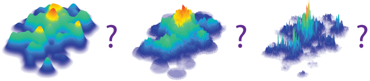


Modeling the initial state

Flow is driven by the initial geometry
Final result depends on what we start with



We need a more rigorous understanding
of the initial state and its fluctuations



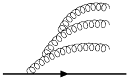
QCD should tell us what the incoming nuclei look like
and what the fluctuations are

$\rightarrow$ use Color Glass Condensate (CGC) effective theory of QCD
and Yang-Mills simulation of two colliding CGCs

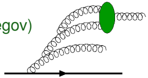
Nuclei at high energy: Gluon saturation

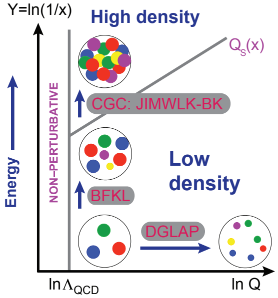
As we go to higher energy / smaller x, gluons split, number increases:

BFKL (Balitsky,Fadin,Kuraev,Lipatov) equation describes x-evolution
but violates unitarity: cross-sections grow without bound



JIMWLK (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner) and BK (Balitsky, Kovchegov)
equations include non-linear evolution $\rightarrow$ saturation





small x: wavefunction characterized by the saturation scale $Q_s(x) \gg \Lambda_{\mathrm{QCD}}$

$\mathbf{p}_T \lesssim Q_s(x)$:

- strong saturated fields $A_\mu \sim 1/g$
- occupation numbers $\sim 1/\alpha_s$
- $\rightarrow$ classical field approximation

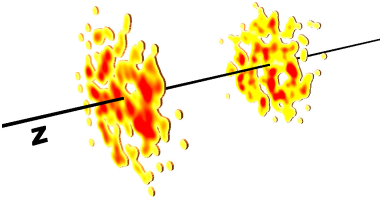
x = longitudinal momentum fraction of partons in a hadron or nucleus

Saturation model for the color charge density

Energy and impact parameter b dependence of $Q_s(x, \mathbf{b})$
can be modeled in the IP-Sat model Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

Parametrize cross sections for DIS on protons
and fit to HERA diffractive data $\rightarrow Q_s(x, \mathbf{b})$

Color charge density $g\mu(x, \mathbf{b})$ is proportional to $Q_s(x, \mathbf{b})$
For a nucleus sample nucleon positions and add all $g^2\mu^2(x, \mathbf{x}_\perp)$



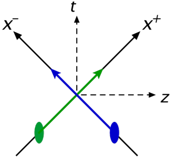
Color charges and gluon fields

Sample color charges ρ^a from local Gaussian for each nucleus

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

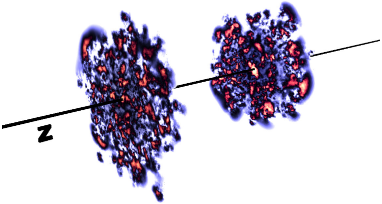
Color charges determine incoming color currents:

$$J_1^\nu = \delta^{\mu+}\rho_1(x^-, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$



$$J_2^\nu = \delta^{\mu-}\rho_2(x^+, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

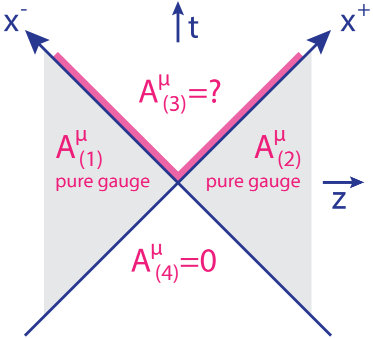
Solve Yang-Mills equations for the gauge fields A^μ



Wilson line correlator shows degree of fluctuations in the gluon fields:
Fluctuation scale: $1/Q_s$

IP-Glasma: Gauge fields after the collision

Initial condition on the lightcone:



Configuration in Schwinger gauge $A^\tau = 0$

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

We solve for the gauge fields numerically

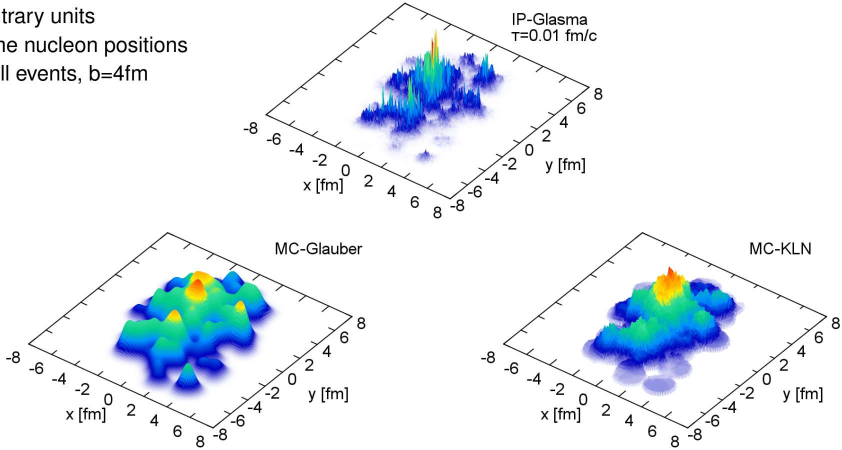
Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

Time evolution follows Yang-Mills equations

Energy density B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett. 108, 252301 (2012)

Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution

arbitrary units
same nucleon positions
in all events, b=4fm



MC-KLN: Drescher, Nara, nucl-th/0611017
mckln-3.52 from `http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html` with defaults, energy density scaling

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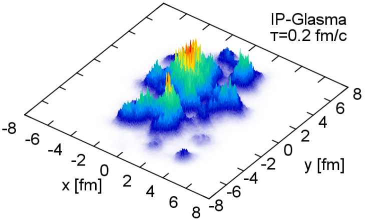

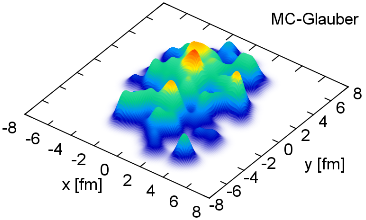

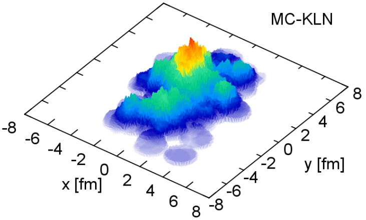

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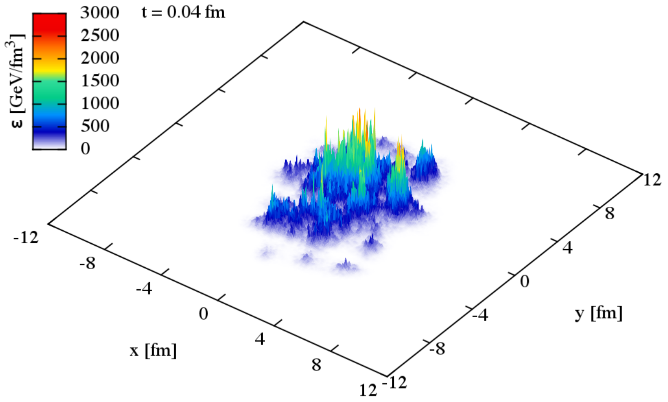
Yang-Mills + viscous hydro evolution

Energy density and initial flow velocity from $u_\mu T^{\mu\nu}_{\mathrm{YM}} = \varepsilon u^\nu$ as input for hydrodynamic simulation



Yang-Mills evolution

Yang-Mills + viscous hydro evolution

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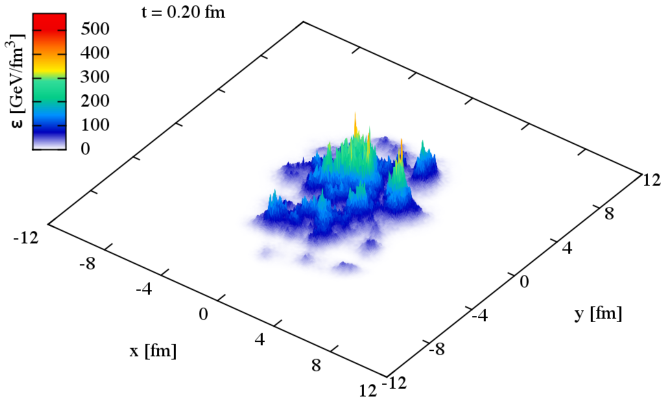
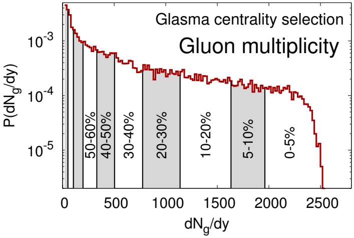


Viscous hydrodynamic evolution

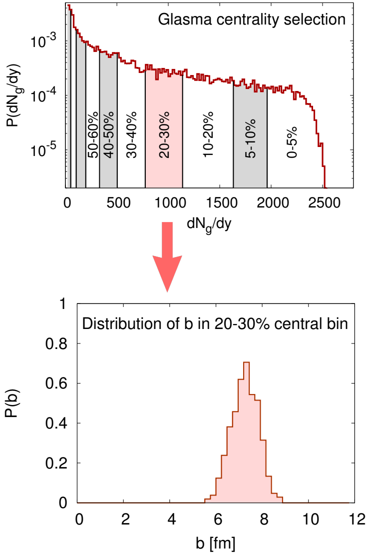
Viscous flow at LHC

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

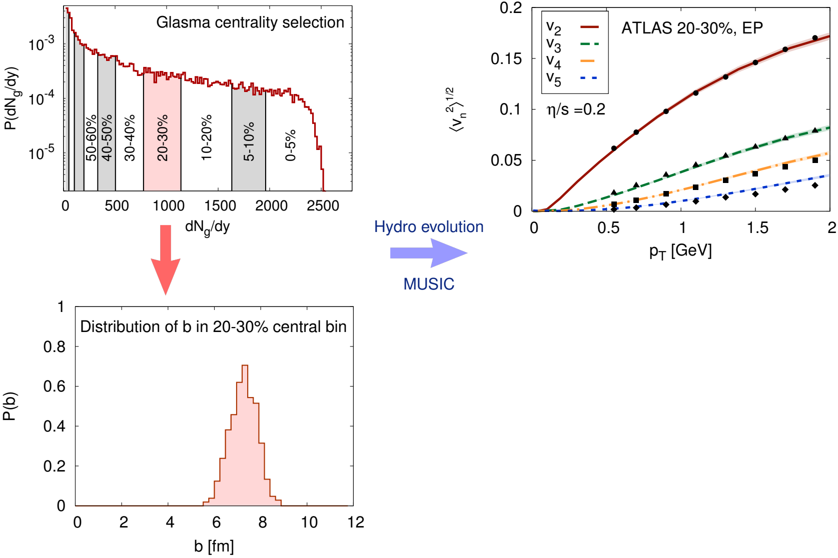


Viscous flow at LHC

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

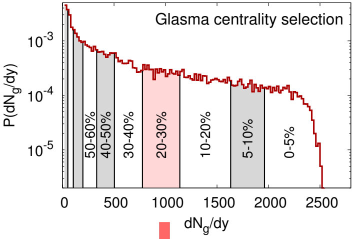


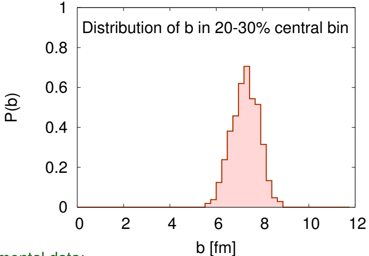
Viscous flow at LHC C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)



Viscous flow at LHC

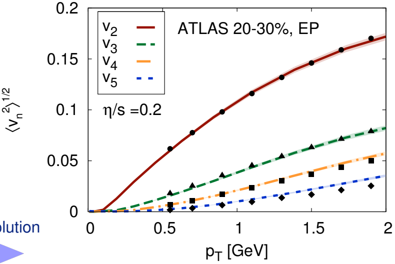
C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

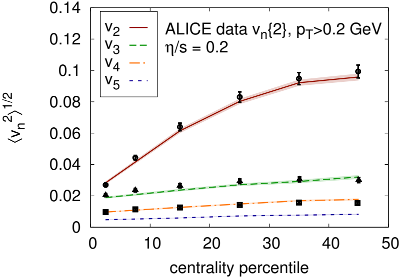




Hydro evolution

MUSIC



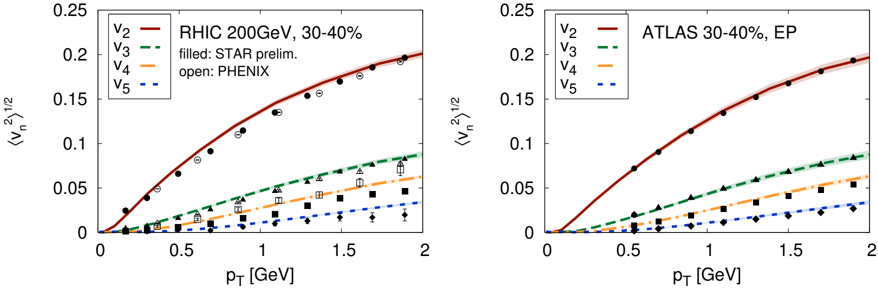


Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)

Viscous flow at RHIC and LHC

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

RHIC $\eta/s = 0.12$ LHC $\eta/s = 0.2$



Results independent of switching time (checked $\tau_{\rm switch} = 0.2$ and 0.4 fm)

Lower effective η/s at RHIC than at LHC needed to describe data

Hints at increasing η/s with increasing temperature

Can be used to gain information on $(\eta/s)(T)$

Experimental data:

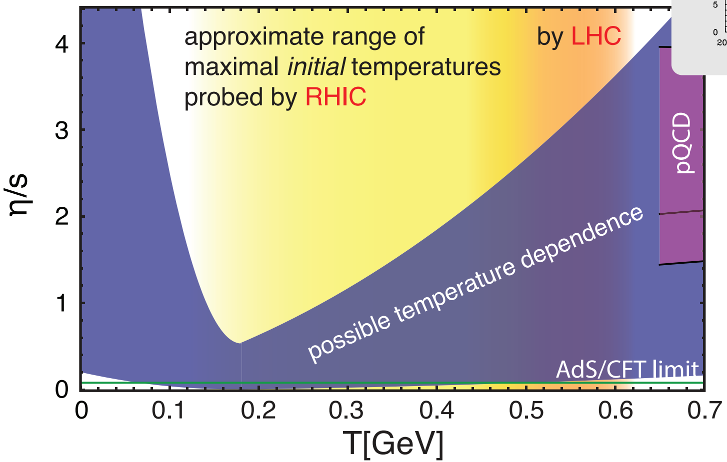
A. Adare et al. (PHENIX Collaboration), Phys.Rev.Lett. 107, 252301 (2011)

Y. Pandit (STAR Collaboration), Quark Matter 2012, (2012)

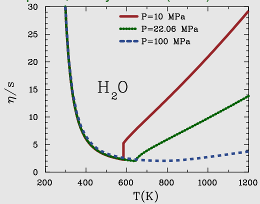
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

Learning about QCD

Example: extraction of $(\eta/s)(T)$



Kapusta, J.Phys. G34 (2007) S295



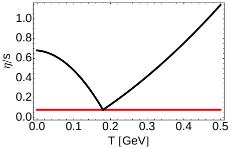
also: bulk viscosity, critical point, and other features

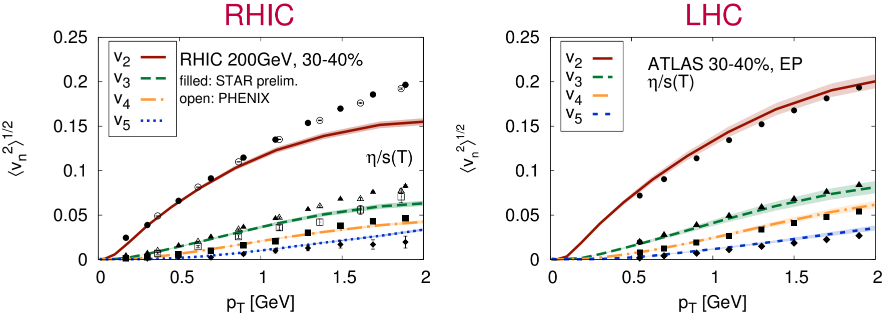
Temperature dependent η/s

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)

Use $\eta/s(T)$ as in Niemi et al., Phys.Rev.Lett. 106 (2011) 212302

Experimental data: A. Adare et al. (PHENIX), Phys.Rev.Lett. 107, 252301 (2011)
Y. Pandit (STAR), Quark Matter 2012, (2012)
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)



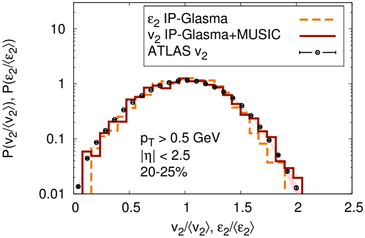


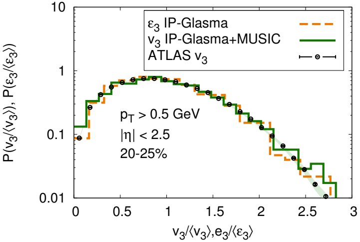
One $(\eta/s)(T)$ will be able to describe both RHIC and LHC data
Used parametrization not yet perfect: no surprise
More detailed study needed - include different RHIC energies and LHC

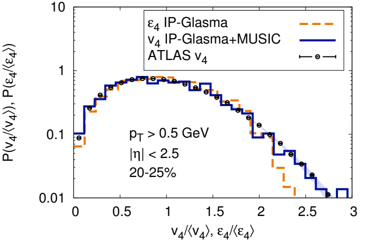
Event-by-event distributions of v_n

https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/

C. Gale, S. Jeon, B.Schenke, P.Tribedy, R.Venugopalan, PRL110, 012302 (2013)







Eccentricities describe v_n distributions well

Excellent agreement with IP-Glasma + flow

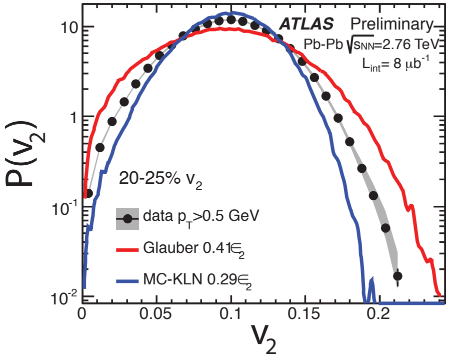
Summary and conclusions

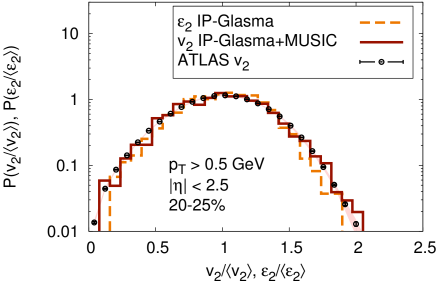
- Hydrodynamics: very successful in describing heavy-ion collisions
- Anisotropic flow: sensitive to fluctuating initial state and transport parameters
- QCD-based model for the initial state: IP-Glasma
- Flow at LHC and RHIC is well described by IP-Glasma + MUSIC
- Fundamental theory combined with extensive numerical efforts and phenomenology $\rightarrow$ quantitative information on fundamental properties of QCD under extreme conditions
- Extend studies to p+A, d+A, lower energy HICs (beam energy scan at RHIC, FAIR)

BACKUP

Event-by-event distributions of v_n - other models

Showing eccentricity distributions (yellow on the right)



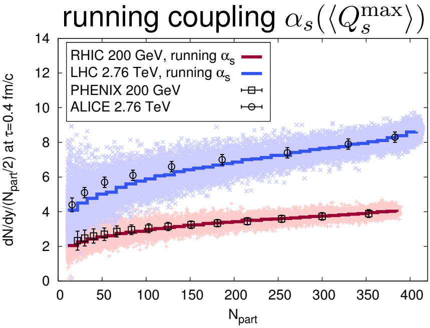


Event-by-event distributions can distinguish between different initial state models

Experimental data: `https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/`

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

dN_{g}/dy in transverse Coulomb gauge $\partial_i A^i = 0$
N_{part} from MC-Glauber with $\sigma_{NN} = 42\,\mathrm{mb}$ and $64\,\mathrm{mb}$ respectively

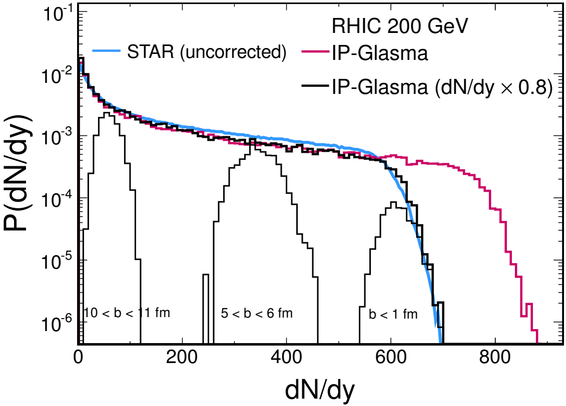


Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles
Normalized to RHIC data

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, Phys. Rev. C86, 034908 (2012)

Experimental data: STAR, Phys. Rev. C79, 034909 (2009)



Glasma model gives a convolution of negative binomial distributions

No need to put them in by hand

Yang-Mills evolution

The equations of motion are formulated on the lattice in 2+1D ($\phi = A_\eta$)

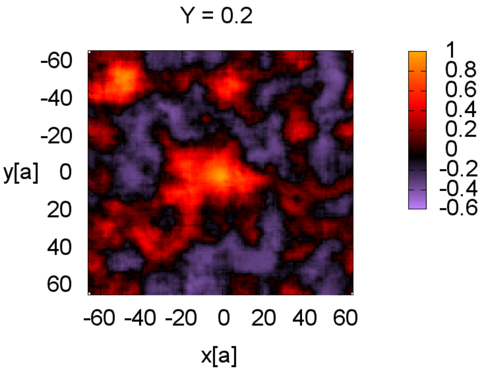
$$\dot{U}_i = i\frac{g^2}{\tau}E^i U_i \text{ (no sum over } i\text{)}$$
$$\dot{\phi} = \tau E^\eta$$
$$\dot{E}^1 = \frac{i\tau}{2g^2}[U_{1,2} + U_{1,-2} - U^\dagger_{1,2} - U^\dagger_{1,-2} - T_1] + \frac{i}{\tau}[\tilde{\phi}_1, \phi]$$
$$\dot{E}^2 = \frac{i\tau}{2g^2}[U_{2,1} + U_{2,-1} - U^\dagger_{2,1} - U^\dagger_{2,-1} - T_2] + \frac{i}{\tau}[\tilde{\phi}_1, \phi]$$
$$\dot{E}^\eta = \frac{1}{\tau}\sum_i \left[\tilde{\phi}_i + \tilde{\phi}_{-i} - 2\phi\right]$$

where $T_1 = \frac{1}{N_c}\mathrm{tr}[U_{1,2} + U_{1,-2} - U^\dagger_{1,2} - U^\dagger_{1,-2}]\,\mathbf{1}$, and
$T_2 = \frac{1}{N_c}\mathrm{tr}[[U_{2,1} + U_{2,-1} - U^\dagger_{2,1} - U^\dagger_{2,-1}]\,\mathbf{1}$ with the $N_c \times N_c$ unit matrix $\mathbf{1}$
$\tilde{\phi}^j_i = U^i_j \phi_{j+\hat{e}_i} U^{i\dagger}_j$ and $U^j_{1,2} = U^1_j\, U^2_{j+\hat{e}_1}\, U^{1\dagger}_{j+\hat{e}_2}\, U^{2\dagger}_j$

JIMWLK renormalization group equation: Solution

The JIMWLK equations can be solved numerically after rewriting into functional Boltzmann-Langevin equation (in rapidity)

JIMWLK governs evolution of multi-point Wilson-line correlators
Example: $C(x,y) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V^\dagger(0,0)V(x,y))]$



also useful for calculations of multi-hadron correlators at forward rapidity in d+A or p+A collisions that are sensitive to four- and six-point gluon correlators

A. Dumitru, J. Jalilian-Marian, T. Lappi, B. Schenke, R. Venugopalan
Phys. Lett. B706, 219-224 (2011)

Equations to be solved

Explicit version of $\partial_\mu T^{\mu\nu} = 0$ in an appropriate coordinate system that expands longitudinally

proper time $\tau = \sqrt{t^2 - z^2}$, space-time rapidity $\eta_s = \frac{1}{2}\ln\frac{t+z}{t-z}$

$$\partial_\tau(\tau T^{\tau\tau}) + \partial_v(\tau T^{v\tau}) + \partial_{\eta_s}(T^{\eta_s\tau}) + T^{\eta_s\eta_s}$$
$$+\partial_\tau(\tau\pi^{\tau\tau}) + \partial_v(\tau\pi^{v\tau}) + \partial_{\eta_s}(\pi^{\eta_s\tau}) + \pi^{\eta_s\eta_s} = 0$$

$$\partial_\tau(\tau T^{\tau\eta_s}) + \partial_v(\tau T^{v\eta_s}) + \partial_{\eta_s}(T^{\eta_s\eta_s}) + T^{\tau\eta_s}$$
$$+\partial_\tau(\tau\pi^{\tau\eta_s}) + \partial_v(\tau\pi^{v\eta_s}) + \partial_{\eta_s}(\pi^{\eta_s\eta_s}) + \pi^{\tau\eta_s} = 0$$

$$\partial_\tau(\tau T^{\tau v}) + \partial_w(\tau T^{wv}) + \partial_{\eta_s}(T^{\eta_s v})$$
$$+\partial_\tau(\tau\pi^{\tau v}) + \partial_w(\tau\pi^{wv}) + \partial_{\eta_s}(\pi^{\eta_s v}) = 0$$

spatial derivatives (computed using Kurganov-Tadmor method)

treated as sources

Equations to be solved

In addition, we have to solve the equation for $\pi^{\mu\nu}$
(2^{nd} order Israel-Stewart viscous fluid-dynamics)

$$\begin{aligned}\partial_c(u^c\pi^{ab}) = &-\frac{1}{2\tau}u^\tau\pi^{ab} + \frac{1}{\tau}\Delta^{a\eta}u^\eta\pi^{b\tau} - \frac{1}{\tau}\Delta^{a\tau}u^\eta\pi^{b\eta} \\ &- g_{cf}\pi^{cb}u^a Du^f - \frac{\pi^{ab}}{2\tau_\pi} - \frac{1}{6}\pi^{ab}\partial_c u^c \\ &+ \frac{\eta}{\tau_\pi}\left(-\frac{1}{\tau}\Delta^{a\eta}g^{b\eta}u^\tau + \frac{1}{\tau}\Delta^{a\eta}g^{b\tau}u^\tau \right. \\ &\qquad\left. + g^{ac}\partial_c u^b - u^a Du^b - \frac{1}{3}\Delta^{ab}\partial_c u^c\right) \\ &+ (a \leftrightarrow b)\,,\end{aligned}$$

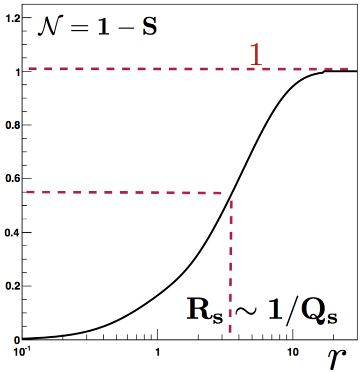
relaxation time τ_π
$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$
$D = u^\mu\partial_\mu$
$\tau_\pi = 3\eta/(\varepsilon + P)$: shear relaxation time

Modeling x and b dependence of Q_s: IP-Sat model

After fitting parameters to HERA DIS data the model provides a distribution of $Q_s^2(x, \mathbf{b})$ of the proton, which will be our input

It is determined self consistently from the requirement that

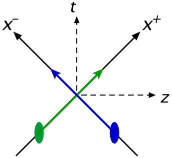
$$\mathcal{N}(R_s, x, \mathbf{b}) = 1 - e^{-1/2}, \quad \text{with } Q_s^2 = \frac{2}{R_s^2}$$



IP-Glasma: 2. Gauge fields before the collision

Color currents:

$$J_1^\nu = \delta^{\mu +}\rho_1(x^-, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_1^\nu$$



$$J_2^\nu = \delta^{\mu -}\rho_2(x^+, \mathbf{x}_\perp)$$
$$[D_\mu, F^{\mu\nu}] = J_2^\nu$$

Solution in covariant gauge:

$$A^+_{\mathrm{cov}(1,2)}(x^-, \mathbf{x}_\perp) = -\frac{g\rho_{(1,2)}(x^-, \mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}$$

with infrared cutoff m of order Λ_{QCD}.

Solution in light cone gauge:

$$A^+_{(1,2)}(\mathbf{x}_\perp) = A^-_{(1,2)}(\mathbf{x}_\perp) = 0$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

V is the path-ordered exponential of $A^+_{\mathrm{cov}(1,2)}$

IP-Glasma: 2. Gauge fields before the collision

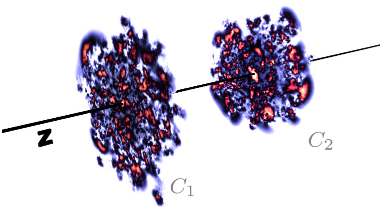
The correlator of the Wilson lines (same one we looked at before)

$$C_{(1,2)}(\mathbf{x}_\perp) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))]$$

with

$$V_{(1,2)}(\mathbf{x}_\perp) = P\exp\left(-ig\int dx^- \frac{\rho_{(1,2)}(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}\right)$$

shows the degree of correlations and fluctuations in the gluon fields.



The length scale of fluctuations is $1/Q_s$. Not the nucleon size.

IP-Glasma: 3. Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

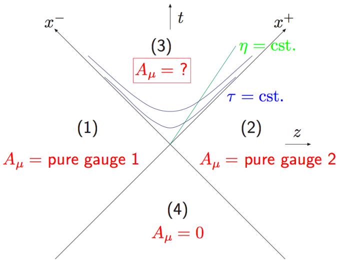


figure from Lappi, arXiv:1003.1852

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

$$\mathrm{tr}\left\{t^a\left[\left(U^i_{(1)} + U^i_{(2)}\right)\left(1 + U^{i\dagger}_{(3)}\right) - \left(1 + U^i_{(3)}\right)\left(U^{i\dagger}_{(1)} + U^{i\dagger}_{(2)}\right)\right]\right\} = 0$$

where t^a are the generators of $SU(N_c)$ in the fundamental representation. Solve iteratively.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

$$U^i_{(1,2),j} = V_{(1,2),j}V^\dagger_{(1,2),j+\hat{e_i}}$$ (gauge transform of $\mathbf{1}$: pure gauge)

E_η can be obtained from $U^i_{(1,2,3)}$

IP-Glasma: Initial energy density

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett.108, 252301 (2012)

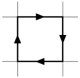
Initial energy density at $\tau = 0$:

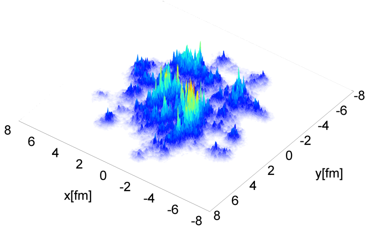
$$\varepsilon(\tau = 0) = \frac{2}{g^2 a^4}(N_c - \mathrm{Re}\,\mathrm{tr}\,U_\Box) + \frac{1}{a^4}\mathrm{tr}\,E_\eta^2$$

with the longitudinal magnetic and electric energy density.

The plaquette is given by

$$U_\Box^j = U_{1,2}^j = U_j^1\, U_{j+\hat{e}_1}^2\, U_{j+\hat{e}_2}^{1\dagger}\, U_j^{2\dagger}$$





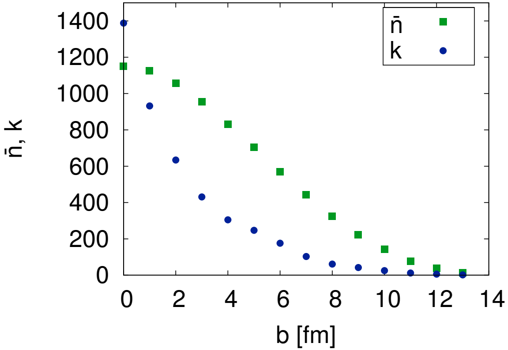
Negative binomial fluctuations

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

Extract k and $\bar{n}$ using a fit with

$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

at fixed impact parameters

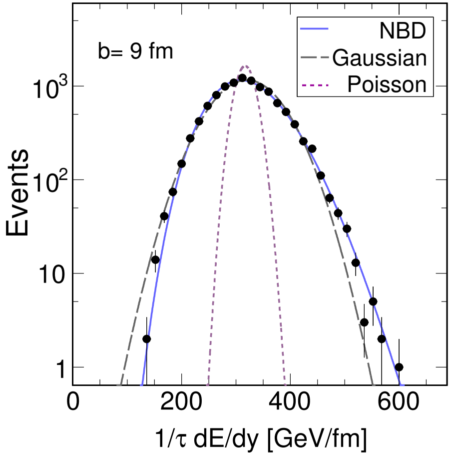


Ratio of $k/\bar{n}$ is > 1 for small b and becomes small ~ 0.14 for large b.
That is close to the value extracted for $p+p$ collisions: Dumitru and Nara arXiv:1201.6382

Negative binomial fluctuations

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett.108, 252301 (2012)

Fluctuations in the total energy per unit rapidity produce negative binomial distribution (NBD).



$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

Good, since multiplicity in pp collisions can be described well with NBD.

In AA, convolution of NBDs at all impact parameters describes data well too.

P. Tribedy and R. Venugopalan
Nucl.Phys. A850 (2011) 136-156

MC-KLN does not do that - these fluctuations need to be put in by hand.

see Dumitru and Nara arXiv:1201.6382

NBDs and Glasma flux tubes

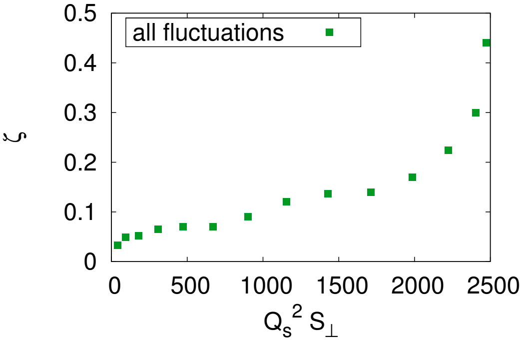
B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

Glasma flux tube picture:

$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

Gelis, Lappi, Mclerran, arXiv:0905.3234

Width of NBD is inversely proportional to the number of flux tubes $Q_s^2 S_\perp$.
$S_\perp$ = interaction area



ζ should be close to constant in the flux tube picture

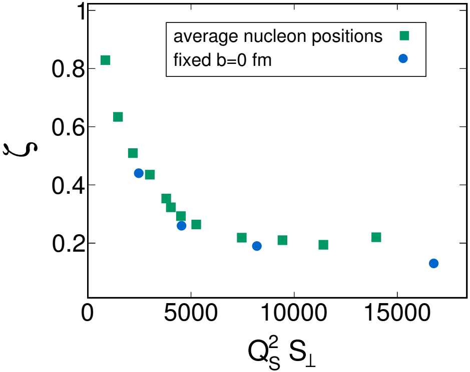
NBDs and Glasma flux tubes

B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)

ζ is not constant because geometric fluctuations are very important
Were not considered in the derivation of

$$k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp$$

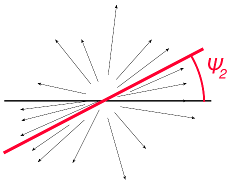
Eliminate by using smooth nucleon distributions:



Event plane

To get non-zero odd moments, we rotate the event plane in each event.
Event plane is defined by the angle:

$$\psi_n = \frac{1}{n} \arctan \frac{\langle w \sin(n\phi) \rangle}{\langle w \cos(n\phi) \rangle}$$



using particle momenta.
($w = p_T$ (first results) or 1 (later results))

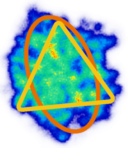
A.Poskanzer and S.Voloshin, Phys.Rev.C58:1671-1678 (1998)

$$v_n = \langle \cos(n(\phi - \psi_n)) \rangle$$

... different angle for every flow coefficient.

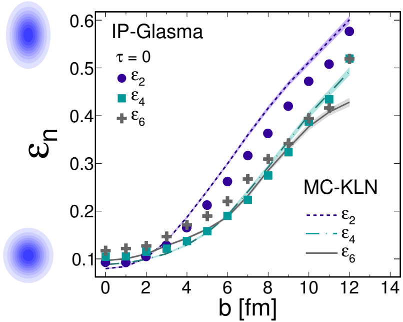
In the simulation we know the true event-plane (no correction factor).

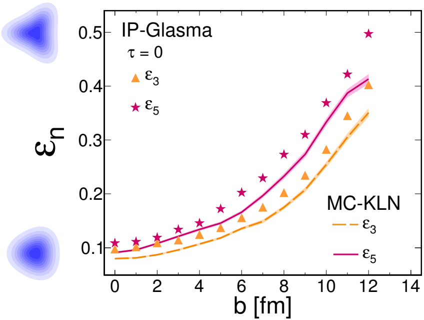
Eccentricities B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.C86, 034908 (2012)



Characterize the initial distribution
by its ellipticity, triangularity, etc...

$$\varepsilon_n = \sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2} / \langle r^n \rangle$$





- ε_n larger in Glasma model for odd n
- ε_n smaller in Glasma model for $n = 2$ (for $b > 3\,\mathrm{fm}$)
  about equal for $n = 4$, larger for $n = 6$

KLN, fKLN, MC-KLN, ...

- original KLN:
  - uses k_T-factorization
  - $(Q_s^A)^2(\mathbf{x}_\perp) \propto N_{\mathrm{part,A}}(\mathbf{x}_\perp)$.
  - Saturation scales are not universal: $N_{\mathrm{part,A}}(\mathbf{x}_\perp)$ depends on nucleus B.
  - The energy density ($\epsilon \propto Q_{s,\mathrm{larger}} Q^2_{s,\mathrm{smaller}}$) is suppressed in the edge region along the impact parameter direction $\rightarrow$ larger eccentricity.
- fKLN:
  - uses k_T-factorization
  - Different definition of unintegrated gluon distribution (correct limit: where there is one nulceon at the edge the uGDF is that of one nucleon - not so in KLN)
  - Universal saturation scales in nucleus A and B. (Important at the edges of the nuclei)
- MC-KLN: Monte-Carlo implementation of fKLN with fluctuating positions of the nucleons
- IP-Glasma (CYM):
  - Does not use k_T-factorization (because it is strictly not valid in A+A collisions - at least one source has to be dilute)
  - $Q_s(\mathbf{x}_\perp)$ universal and constrained by HERA data.
  - No utilization of the nucleon-nucleon cross section.
  - Takes into account non-linearities.
  - Includes fluctuations of color charges within a nucleon.

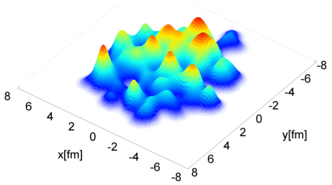
Comparison with other models: “MC-Glauber”

- Sample nucleon positions in nucleus A and B, then overlap the two distributions.
- An interaction happens when the distance d between a nucleon from nucleus A and one from nucleus B fulfills:

$$d \leq \sqrt{\sigma_{\mathrm{inel}}^{NN} / \pi}$$

- Add Gaussian energy density with width σ_0 for every wounded nucleon, binary collision, or combination.

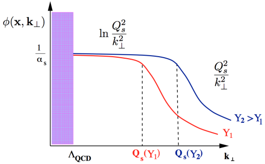
Result for $\sigma_0 = 0.4\,\mathrm{fm}$



Comparison with other models: “MC-KLN”

- Determine gluon production using k_T-factorization:

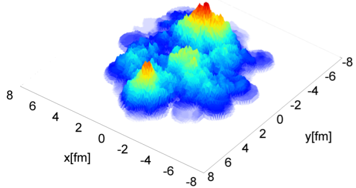
$$\frac{dN}{dr_\perp^2\,dy} \sim \int \frac{d^2p_\perp}{p_\perp^2} \int d^2k_\perp \alpha_s \phi_A(x_1, k_\perp^2)\phi_B(x_2, (p_\perp - k_\perp)^2)$$



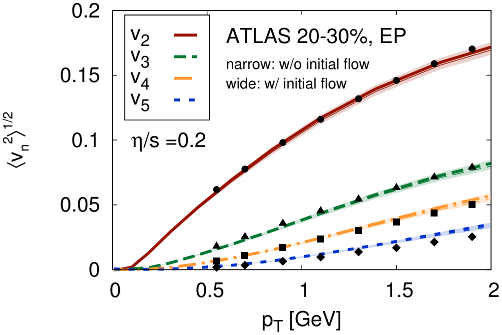
- Energy density analogously.
- Kharzeev-Levin-Nardi (KLN) model for ϕ:

$$\phi_{A,B}(x, k_\perp^2, \mathbf{r}) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2}{\max(Q_s^2, k_\perp^2)}$$

with $Q_{s,A}^2(x, \mathbf{r}) = 2\,\mathrm{GeV}^2 \frac{T_A(\mathbf{x}_\perp)}{1.53\,\mathrm{fm}^{-2}} \left(\frac{0.01}{x}\right)^\lambda, \lambda = 0.28$



Effect of initial flow



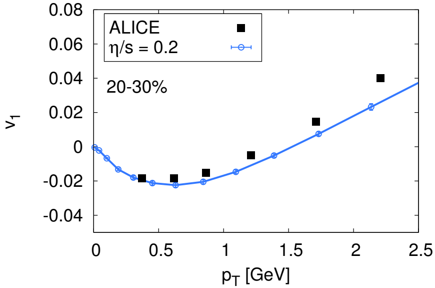
Weak effect of initial flow on hadron $v_n(p_T)$

Expect stronger effect for photon v_n:
Photons are mostly produced early at high temperatures

Effect of different switching time $0.4\,\mathrm{fm}/c$ is very weak

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

Directed flow v_1

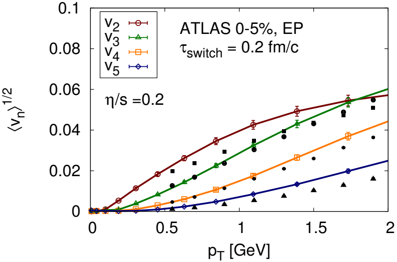


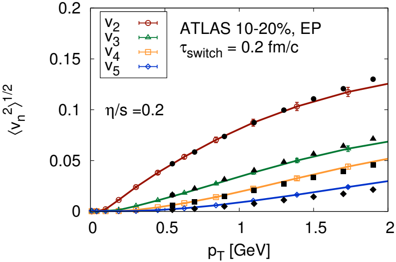
Experimental data:

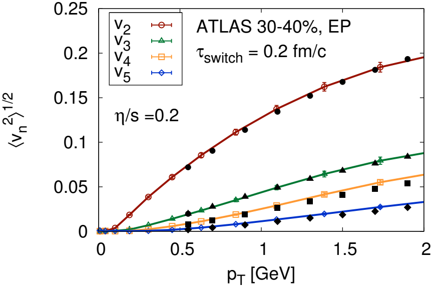
extracted in Retinskaya et al., Phys.Rev.Lett. 108 (2012) 252302

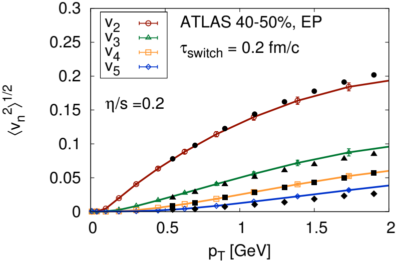
from ALICE data in K. Aamodt et al., Phys. Lett. B 708, 249 (2012)

More centrality classes: IP-Glasma + MUSIC

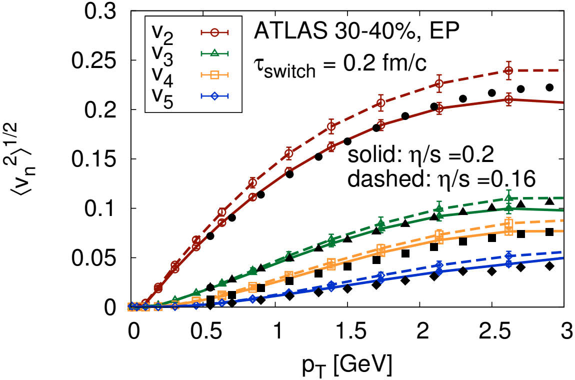








Smaller average η/s



Using $\eta/s = 0.16$ overestimates all v_n

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)