

# Matrix Elements for Fundamental Symmetries

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# Collaborators



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A. Poves



E.Caurier, F. Nowacki



# Outline

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Why testing Fundamental Symmetries with Nuclei?

Nuclear Structure calculations

Derivation of Hadronic Currents: chiral EFT

Nature of Neutrinos and Lepton-number Violation:  
Neutrinoless Double-Beta Decay

Search for Supersymmetric Dark Matter Particles:  
WIMP Scattering off Nuclei

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# Symmetries in Physics

Symmetries play a crucial role in our understanding of Nature

They can lead to **conservation laws**:

- Translational symmetry  $\Rightarrow$  Energy-Momentum conservation
- Rotational symmetry  $\Rightarrow$  Angular Momentum conservation

They explain **properties of particles**:

- Chiral symmetry of QCD  $\Rightarrow$  Pions are much lighter than other mesons

Some symmetries predict **allowed processes** (unless violated)

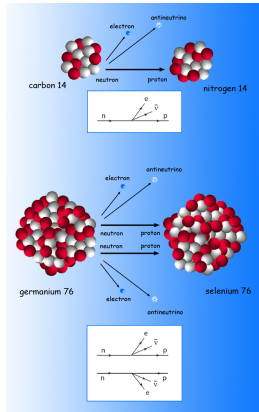
- Lepton number (L), Baryon number (B), CPT...

And there is **new symmetries** that have been proposed:

- Supersymmetry...

# Lepton-Number (L) conservation

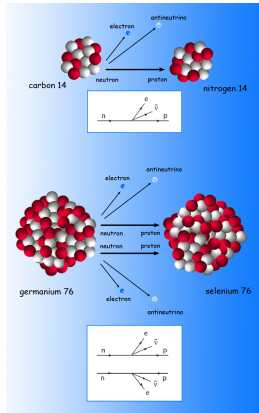
Lepton number conserved in  
all processes observed so far:



$\beta$  decay,  $\beta\beta$  decay...

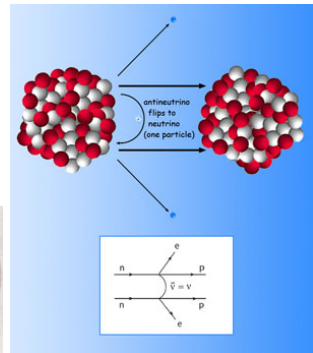
# Lepton-Number (L) conservation

Lepton number conserved in all processes observed so far:



$\beta$  decay,  $\beta\beta$  decay...

Uncharged massive particles like Majorana neutrinos ( $\nu$ ) theoretically allow L violation



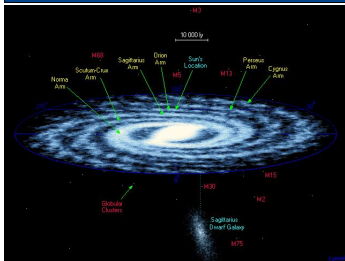
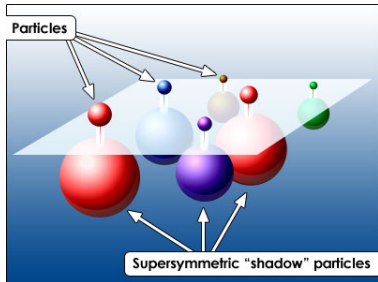
Neutrinoless  $\beta\beta$  ( $0\nu\beta\beta$ ) decay

# Supersymmetry (SUSY)

Supersymmetry  
(supersymmetric partners for the  
fundamental particles we know)  
introduced in extensions of the  
Standard Model of particle physics  
They solve purely theoretical  
problems (gauge hierarchy)

Supersymmetric particles  
promising candidates  
for Dark Matter of our Universe

They would naturally occur in  
observed Dark Matter densities



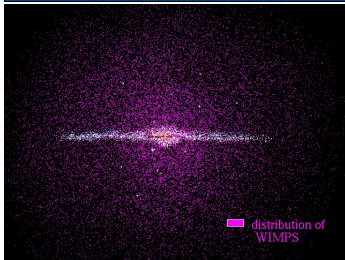
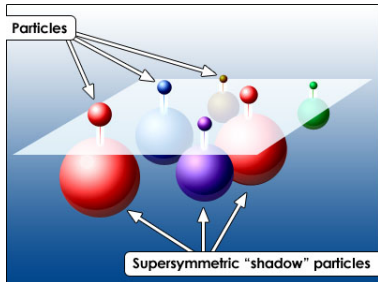


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# Testing Symmetries with Nuclei

Symmetries can be test in particle accelerators (LHC)

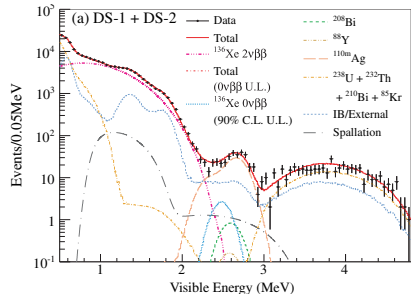
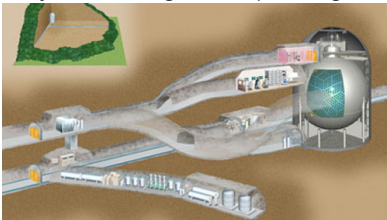
Experiments with nuclei offer an alternative:

Nuclei are abundant in huge numbers  $N_A = 6.02 \cdot 10^{23}$  nuclei in A grams!

Lots of material over long times allow to:

- Detect L-violating processes with very small branching ratios
- Measure SUSY-baryon cross-sections even if interactions are weak

Isolate from other processes:  
very low background (underground)



KamLAND-Zen, PRL110 062502 (2013)

# Matrix Elements for Fundamental Symmetries

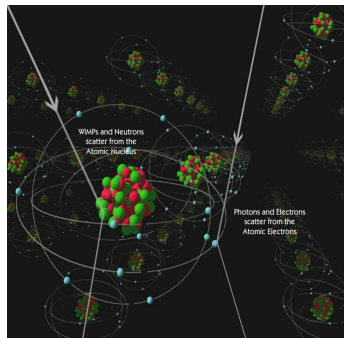
To study Fundamental Symmetries we need Nuclear Matrix Elements

$$\langle \text{Initial} | H_{\text{leptons-nucleons}} | \text{Final} \rangle = \langle \text{Initial} | \int dx j^\mu(x) J_\mu(x) | \text{Final} \rangle$$

- **Nuclear structure calculation of the initial and final states:**

State-of-the-art Shell Model  
diagonalizations and interactions

- **Description of the lepton-nucleus interaction:**  
Evaluation (non-perturbative)  
of the hadronic currents  
inside nucleus



CDMS Collaboration

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Nuclear Structure calculations

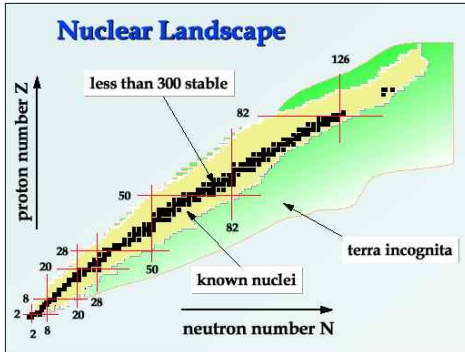
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# Nuclear Structure approach



Big variety of nuclei in the nuclear chart,  $A \sim 2 \dots 300$

Systematic *ab initio* calculations only possible in the lightest nuclei

Poses a hard many-body problem: design approximate methods suited for different regions

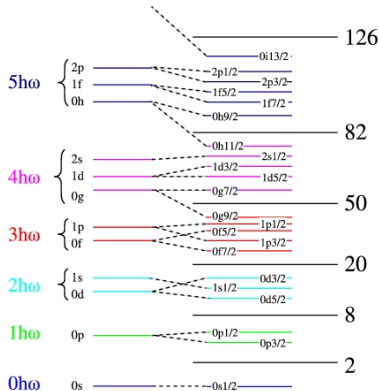
## Interacting Shell Model:

Solve the problem choosing the (more) relevant degrees of freedom

Use realistic nucleon-nucleon (NN) and three-nucleon (3N) interactions

Phenomenological adjustments if necessary

# The Interacting Shell Model



Chose as basis states that of the 3D Harmonic Oscillator

To keep the problem feasible, the configuration space is separated into

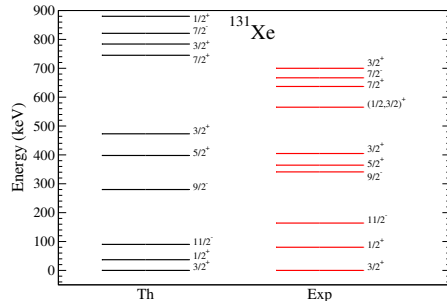
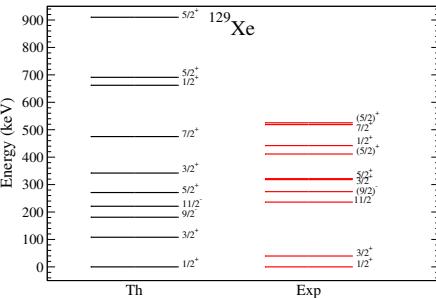
- Inner core: orbits that are always filled
- Valence space: the space in which we explicitly solve the problem
- Outer space: orbits that are always empty

Solve in valence space:  $H|\Psi\rangle = E|\Psi\rangle \rightarrow H_{eff}|\Psi\rangle_{eff} = E|\Psi\rangle_{eff}$

where  $H_{eff}$  is obtained in many-body perturbation theory (MBPT)  
includes the effect of inner core and outer orbits

# $^{129,131}\text{Xe}$ spectra

The agreement with experimental spectra is very good!



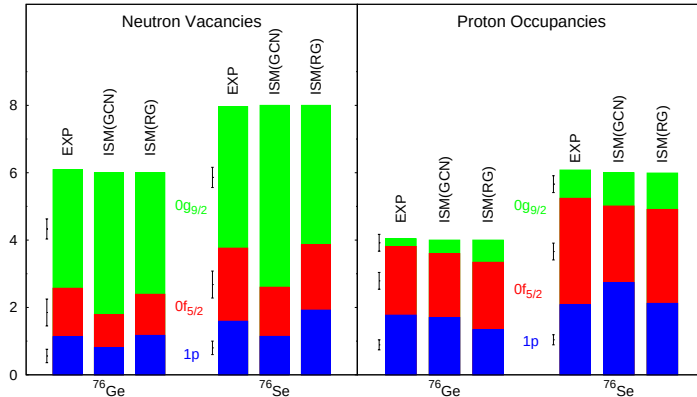
JM, Gazit, Schwenk PRD86 103511(2012)

Note that show first 10 states out of total  $\sim 10^8$

No information about these particular nuclei present in the calculation

# Occupancies of $^{76}\text{Ge}$ , $^{76}\text{Se}$

Experimental occupancies are well described!



Experiment: Schiffer et al. PRL100 112501(2009), Kay et al. PRC79 021301(2009)

Theory: JM, Caurier, Nowacki, Poves PRC80 048501 (2009)





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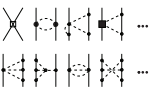
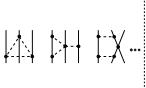
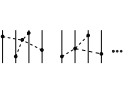
# Forces and Currents in Chiral EFT

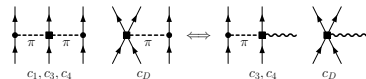
Chiral EFT: low energy approach to QCD for nuclear structure energies

Approximate chiral symmetry of QCD: pions pseudo-Goldstone bosons

Short-range couplings are fitted to experiment once

Systematic expansion: nuclear forces and electroweak currents

|                   | 2N force                                                                          | 3N force                                                                          | 4N force                                                                          |
|-------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| LO                |  | —                                                                                 | —                                                                                 |
| NLO               |  | —                                                                                 | —                                                                                 |
| N <sup>2</sup> LO |  |  | —                                                                                 |
| N <sup>3</sup> LO |  |  |  |



Same NN and 3N couplings

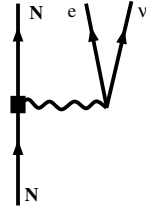
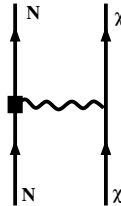
Park et al. PRC67 055206(2003)

Weinberg, van Kolck, Savage, Epelbaum, Kaiser, Meißner...

# Hadronic currents in chiral EFT

At lowest orders  $Q^0$  and  $Q^2$   
there is one-body (1b) currents

Same expressions obtained  
using phenomenological  
arguments

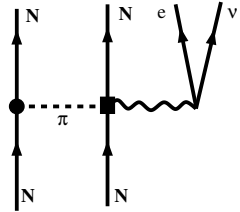
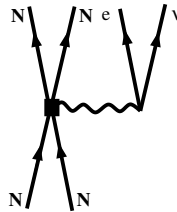


At order  $Q^3$  chiral EFT  
predicts contributions from  
two-body (2b) currents

Reflect interactions  
between nuclei

Long-range currents dominant  
Relevant momentum transfers

$\sim m_\pi$



# Two-body currents in light nuclei

Two-body currents needed to reproduce data in light nuclei:

$^3\text{H}$   $\beta$  decay

Gazit, Quaglioni, Navrátil

PRL103 102502(2009)  $\Rightarrow$

$^6\text{He}$   $\beta$  decay

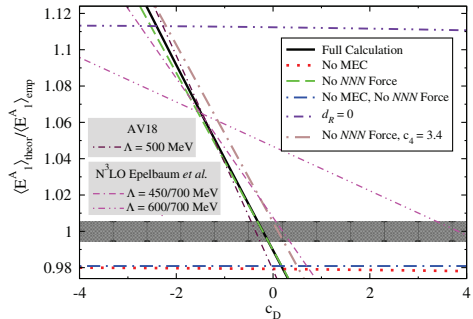
Vaintraub, Barnea, Gazit

PRC79 065501(2009)

$^3\text{H}$   $\mu$  capture

Gazit PLB666 472(2008)

Marcucci et al. PRC83 014002(2011)



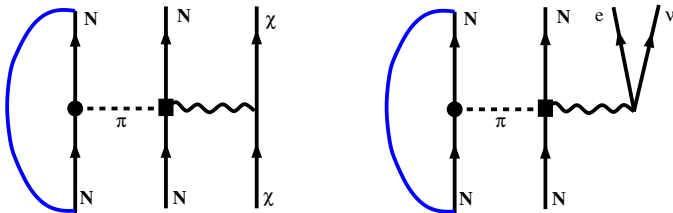
2b current contributions  $\sim$  few % in light nuclei ( $Q \sim \sqrt{BEm}$ )

2b currents order  $Q^3 \Rightarrow$  larger effect in medium-mass nuclei ( $Q \sim k_F$ )

# Normal-ordered 2b currents

Approximate 2b currents in medium-mass nuclei taking normal-ordered 1-body approximation with respect to Fermi gas,

Sum over one nucleon, direct and the exchange terms



$\Rightarrow \mathbf{J}_{n,2b}^{\text{eff}}$ , normal-ordered (effective) one-body current

$\mathbf{J}_{n,2b}^{\text{eff}}$  depends on chiral EFT couplings  $c_3, c_4$  known, with uncertainties, from nuclear force studies

Corrections are  $\sim (n_{\text{valence}}/n_{\text{core}})$  in Fermi systems



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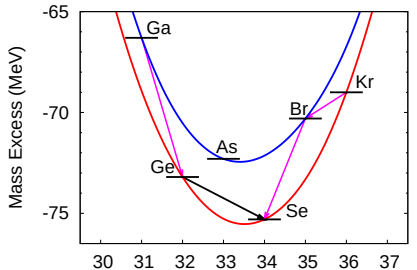
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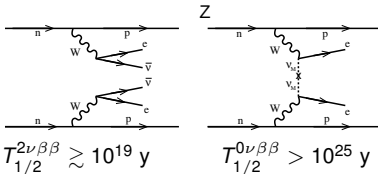


# Neutrinoless Double beta decay

Double beta decay is a second-order process which appears when single- $\beta$  decay is energetically forbidden or hindered by large  $\Delta J$



| Transition                                    | $Q_{\beta\beta}$ (MeV) | Ab. (%) |
|-----------------------------------------------|------------------------|---------|
| $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$   | 4.274                  | 0.2     |
| $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$   | 2.039                  | 8       |
| $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$   | 2.996                  | 9       |
| $^{96}\text{Zr} \rightarrow ^{96}\text{Mo}$   | 3.350                  | 3       |
| $^{100}\text{Mo} \rightarrow ^{100}\text{Ru}$ | 3.034                  | 10      |
| $^{110}\text{Pd} \rightarrow ^{110}\text{Cd}$ | 2.013                  | 12      |
| $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$ | 2.802                  | 7       |
| $^{124}\text{Sn} \rightarrow ^{124}\text{Te}$ | 2.288                  | 6       |
| $^{130}\text{Te} \rightarrow ^{130}\text{Xe}$ | 2.530                  | 34      |
| $^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$ | 2.462                  | 9       |
| $^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$ | 3.667                  | 6       |



Only candidates with  $Q_{\beta\beta} > 2 \text{ MeV}$   
experimentally interesting for  $0\nu\beta\beta$  decay  
(very slow process)

# Nuclear Matrix Elements (NMEs)

$0\nu\beta\beta$  process needs massive Majorana neutrinos ( $\nu = \bar{\nu}$ )  
 $\Rightarrow$  detection would proof Majorana nature of neutrinos

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right)^{-1} = G_{01} |M^{0\nu\beta\beta}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

$M^{0\nu\beta\beta}$  necessary to identify best candidates for experiment and to obtain neutrino masses and hierarchy with  $m_{\beta\beta} = |\sum_k U_{ek}^2 m_k|$

$$M^{0\nu\beta\beta} = \langle 0_f^+ | \sum_{n,m} \tau_n^- \tau_m^- \sum_X H^X(r) \Omega^X | 0_i^+ \rangle$$

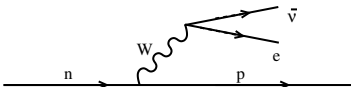
- Many-body method to describe initial and final nuclear states (ISM)
- Transition operator, appropriate for this decay (chiral EFT)



# $M^{0\nu\beta\beta}$ uncertainty: quenching

Major  $M^{0\nu\beta\beta}$  uncertainty is  $g_A$  (quenched?) value:

$$M^{0\nu\beta\beta} \propto g_A^2 \Rightarrow \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4$$



$$\mathbf{J}_{n,1B} = g_A \sigma_n \tau_n^-, \quad g_A^{\text{eff}} = q g_A, \quad q \approx 0.75$$

Theory needs to “quench”  
Gamow-Teller coupling to reproduce  
experimental lifetimes and strength  
functions where the spectroscopy is  
well reproduced

Wildenthal et al. PRC28 1343(1983)

Martínez-Pinedo et al. PRC53 2602(1996)

Bender et al. PRC65 054322(2002)

Rodríguez et al. PRL105 252503(2010)

This puzzle has been the target of many theoretical efforts:

Arima, Rho, Towner, Bertsch and Hamamoto, Wildenthal and Brown...

Revisit in the framework of (chiral EFT) currents

Transferred momenta are high in  $0\nu\beta\beta$  decay:  $p \sim 100 \text{ MeV}$

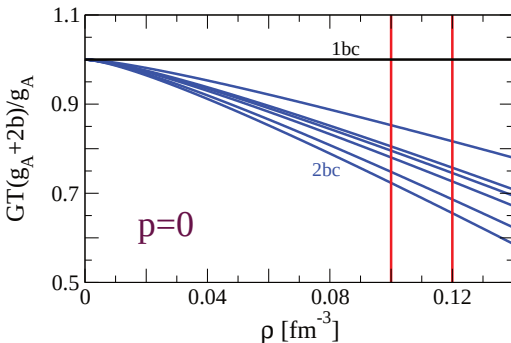
# Contribution of 2b currents

The normal-ordered two-body currents modify GT operator

$$\mathbf{J}_{n,2b}^{\text{eff}} = -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[ \frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} + I(\rho, P) \left( \frac{1}{3} (2c_4 - c_3) + \frac{1}{6m_N} \right) \right],$$

$p$  dependent

long-range



General density range

$$\rho = 0.10 \dots 0.12 \text{ fm}^{-3}$$

Couplings  $c_3, c_4$  from NN potentials

Entem et al. PRC68 041001(2003)

Epelbaum et al. NPA747 362(2005)

Rentmeester et al. PRC67 044001(2003)

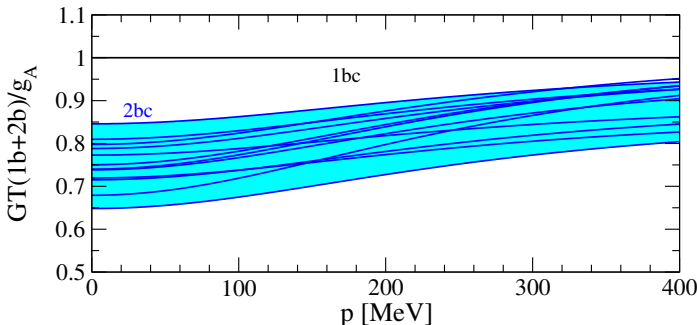
$$\delta c_3 = -\delta c_4 \approx 1 \text{ GeV}^{-1}$$

2b currents predict  $g_A$  quenching

$$q = 0.85 \dots 0.66$$

# Transferred-momentum dependence

The  $\sigma\tau^-$  term depends on transferred momentum  $p$ :  $\frac{2}{3} c_3 \frac{p^2}{4m_\pi^2 + p^2}$

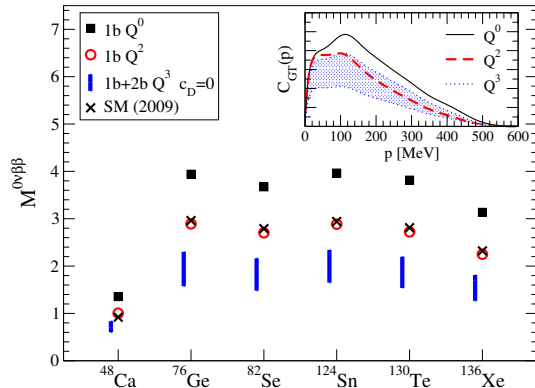


JM, Gazit, Schwenk PRL107 062501 (2011)

Quenching gets weaker at  $p > 0$

Typically  $p \sim 100 \text{ MeV} \sim m_\pi$  for  $0\nu\beta\beta$  decay

# Nuclear Matrix Elements for $0\nu\beta\beta$ decay



JM, Gazit, Schwenk PRL107 062501 (2011)

Calculations using  
state-of-the-art ISM interactions  
and valence spaces  
(NATHAN code)

Order  $Q^0+Q^2$  similar to  
calculations with  
phenomenological currents  
JM, Poves, Caurier, Nowacki  
NPA818 139 (2009)

At order  $Q^3$  2b currents reduce  $\sim 35\%$  the NME

Smaller than  $-45\%$  ( $q^2 = 0.74^2$ ) due to momentum-transfer  $p > 0$

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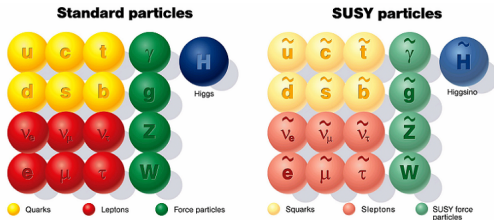
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# SUSY, WIMPs and Dark Matter

Supersymmetric particles proposed in extensions of the Standard Model include Weakly Interacting Massive Particles (WIMPs)

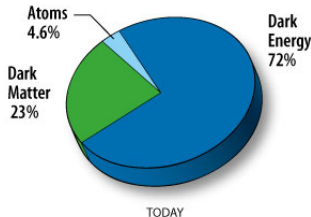


WIMPs are Dark Matter candidates

DM challenge: we know it is there,  
we don't know of what it consists

Evidence for DM strong but indirect:  
rotation curves, lensing...

Motivates DM direct searches



# Spin-Dependent WIMP scattering off nuclei



Spin-Dependent elastic WIMP scattering off nuclei  $\Rightarrow$  odd- $A$  nuclei

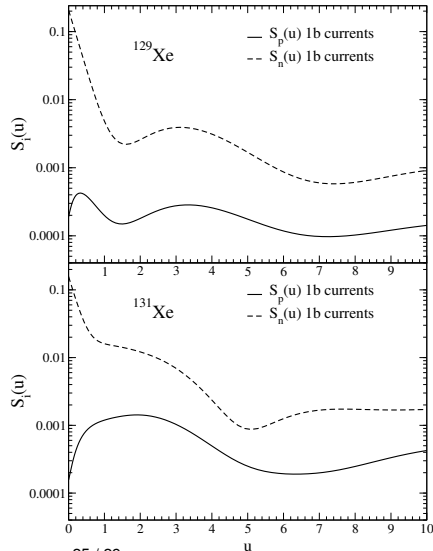
The cross-section is governed by the Structure factor  $S(p)$

$$\frac{d\sigma}{dp} \propto \frac{1}{2} \sum_{s_i, s_f} \frac{1}{2J_i + 1} \sum_{M_i, M_f} |\langle f | H_\chi^{SD} | i \rangle|^2 = G_F^2 \frac{4\pi}{2J_i + 1} \mathbf{S}(\mathbf{p})$$

$\mathbf{S}(\mathbf{p}) = \sum_L \left( |\langle J_f || \mathcal{E}_L(p) || J_i \rangle|^2 + |\langle J_f || \mathcal{L}_L(p) || J_i \rangle|^2 \right)$ , Trans. Electric+Longitudinal multipoles

$$\begin{aligned} \mathcal{E}_L(p) &= \frac{1}{\sqrt{2L+1}} \sum_{i=1}^A \frac{1}{2} \left[ a_0 + a_1 \tau_i^3 \left( 1 - 2 \frac{p^2}{\Lambda_A^2} + \delta a_1 \right) \right] & \delta a_1 < 0 \\ &\times \left[ -\sqrt{L} M_{L,L+1}(pr_i) + \sqrt{L+1} M_{L,L-1}(pr_i) \right] & \delta a_1^P(p) > 0 \\ \mathcal{L}_L(p) &= \frac{1}{\sqrt{2L+1}} \sum_{i=1}^A \frac{1}{2} \left[ a_0 + a_1 \tau_i^3 \left( 1 + \delta a_1 - \frac{2g_{\pi pn} F_\pi p^2}{2mg_A(m_\pi^2 + p^2)} + \delta a_1^P(p) \right) \right] \\ &\times \left[ \sqrt{L+1} M_{L,L+1}(pr_i) + \sqrt{L} M_{L,L-1}(pr_i) \right], \text{ with } M_{L,L'}(pr_i) = j_{L'}(pr_i) [Y_{L'}(\hat{\mathbf{r}}_i) \sigma_i]^L. \end{aligned}$$

# Structure factors with 1b currents



Isoscalar and isovector WIMP-nucleus couplings shape Structure factor

$$S(p) = a_0^2 S_{00} + a_0 a_1 S_{01} + a_1^2 S_{11}$$

In  $^{129,131}_{54}\text{Xe}$   $\langle S_n \rangle \ll \langle S_p \rangle$  and

$$S(0) = \frac{(2J+1)(J+1)}{\pi J} \left| \frac{a_0 + a_1}{2} \langle S_p \rangle + \frac{a_0 - a_1}{2} \langle S_n \rangle \right|^2$$

'Neutron'/'Proton' couplings

$a_1 = -a_0/a_1 = a_0$  maximize/minimize  $S$ :

$$S_n \propto |\langle S_n \rangle|^2 \text{ and } S_p \propto |\langle S_p \rangle|^2$$

Distinction no longer holds at  $p > 0$ :  
peak in  $S_p(u)$ , with  $u = p^2 b^2 / 2$

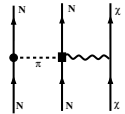
Klos, JM, Gazit, Schwenk, to be submitted



# Structure factors with 1b+2b currents

2b currents allow, in the  $a_1 = a_0$  case, neutrons to contribute at  $p = 0$

$$S(0) \propto \left| \frac{a_0 + a_1(1 + \delta a_1)}{2} \langle S_p \rangle + \frac{a_0 - a_1(1 + \delta a_1)}{2} \langle S_n \rangle \right|^2$$

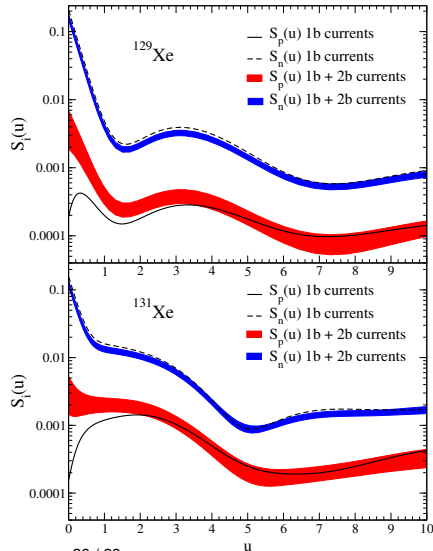


increase dramatically  $S_p(u)$

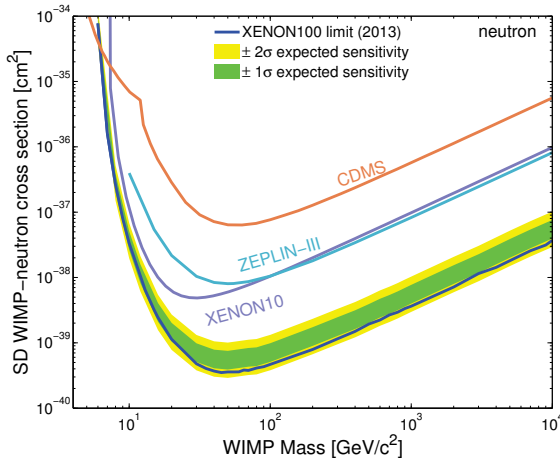
$S_n$  reduced by 10-30% at  $u \sim 1$   
due to  $a_1$  2b correction

$S_n$  possibly enhanced at  $u \gtrsim 3$   
due to  $a_1^P(p)$  2b term

Details depend on relative contributions  
from Trans. Electric/Longitudinal



# Application to experiment



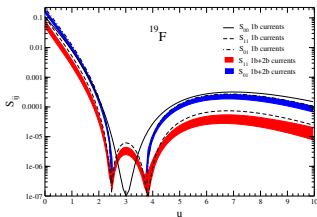
Our calculations used by  
XENON100 Collaboration  
to set limits on WIMP-nucleus  
cross-sections (CS)

XENON100 obtained  
world best limits  
(CS above limit excluded)  
for Spin-Dependent scattering  
with 'neutron' couplings

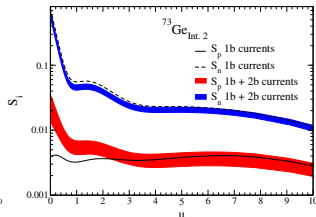
Aprile et al. arXiv:1301.6620



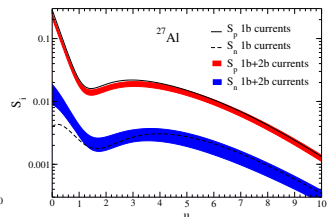
# $^{19}\text{F}$ , $^{23}\text{Na}$ , $^{27}\text{Al}$ , $^{29}\text{Si}$ , $^{73}\text{Ge}$ , $^{127}\text{I}$



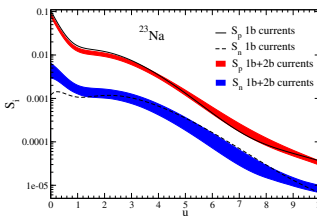
PICASSO, COUPP, SIMPLE



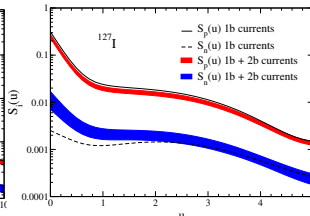
CDMS, EDELWEISS, EURECA



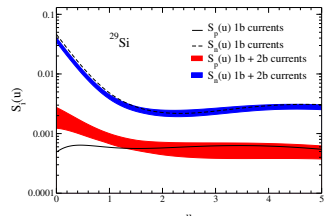
CRESST



DAMA, ANAIS, DM-Ice



DAMA, ANAIS, DM-Ice, KIMS



CDMS-II

# Outline

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Why testing Fundamental Symmetries with Nuclei?

Nuclear Structure calculations

Derivation of Hadronic Currents: chiral EFT

Nature of Neutrinos and Lepton-number Violation:  
Neutrinoless Double-Beta Decay

Search for Supersymmetric Dark Matter Particles:  
WIMP Scattering off Nuclei

Summary



# Summary

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Nuclei ideal to test Fundamental Symmetries: large masses, long times

Matrix Elements key to design experiments and interpret findings

State-of-the-art nuclear structure calculations using ISM:  
good spectroscopic description

Chiral EFT provides hadronic currents including 2b currents

Matrix Elements for Neutrinoless  $\beta\beta$  decay for several candidates

- 2b currents reduce (quench) Matrix Elements in 35%
- $p$ -dependence of GT reduction predicted

Structure factors for Spin-Dependent elastic WIMP scattering off nuclei  
for experimentally relevant nuclei

- Reduce the isovector dominant Structure factor
- Strong increase in sub-dominant Structure factor at low  $p$