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Discrete symmetries in supersymmetric models of flavor

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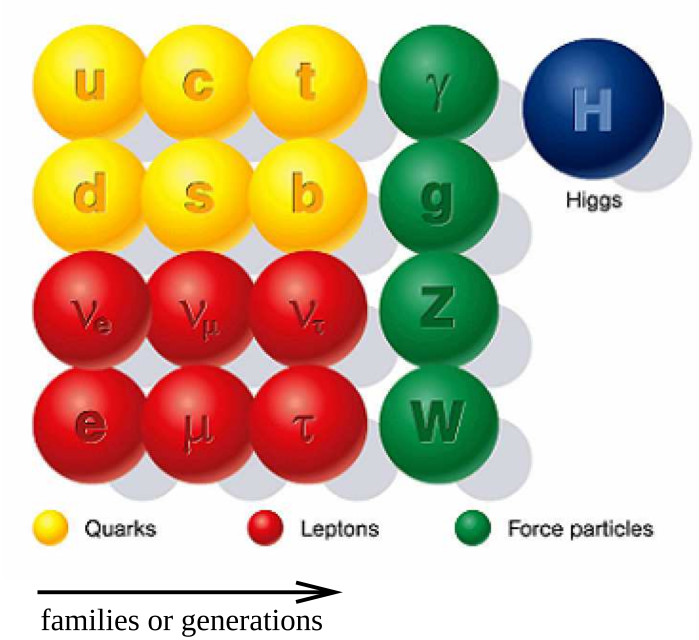
IP3

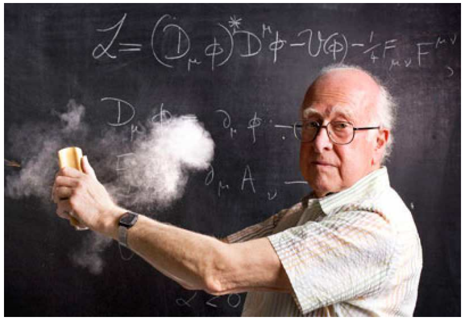
Outline

- supersymmetric Standard Model
  - Z_2 symmetries R-parity and matter parity
  - possible $U(1)_{B-L}$ origin
- Abelian family symmetries
  - $U(1)_{\mathrm{FN}}$ for explaining hierarchies
  - residual discrete symmetries
  - R-symmetries
- non-Abelian family symmetries
  - motivation
  - promising candidates
  - direct and indirect implementation
  - benchmark example based on S_4

Standard Model (of particle physics)



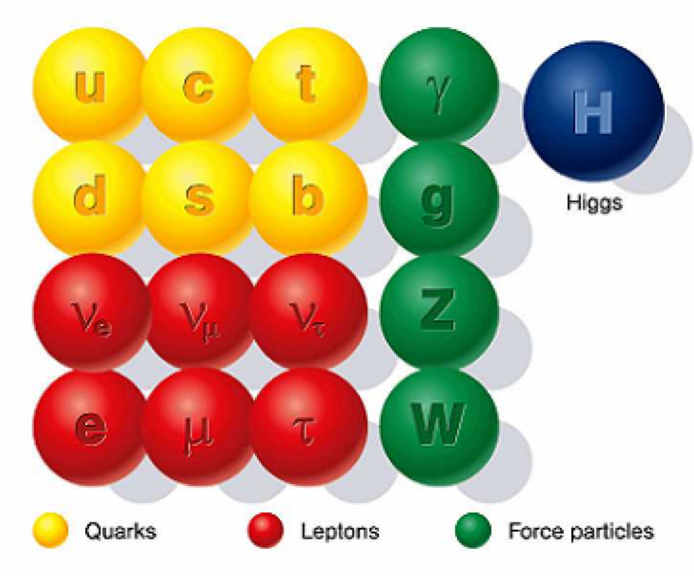
- highly successful theory
- based on gauge symmetry $SU(3)_C \times SU(2)_W \times U(1)_Y$
- broken by Higgs vacuum



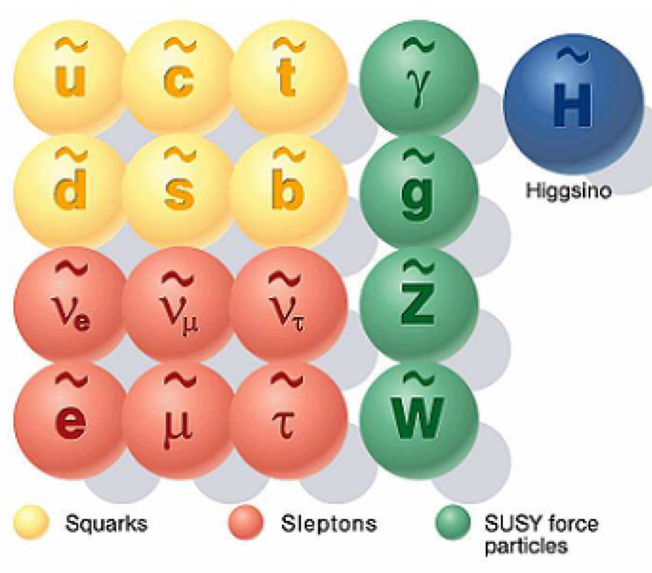
Why look beyond?

- hierarchy problem (protecting the Higgs mass)
- origin of three families of quarks & leptons
- neutrino masses and mixing
- baryogenesis, dark matter, dark energy, etc.

Supersymmetric Standard Model



$\times 2$



$\times 2$

- addresses hierarchy problem
- most general space-time symmetry (extension of Poincaré symmetry)

- supersymmetry must be broken – at TeV scale?
- in total > 300 parameters Barbier et al. (hep-ph/0406039)
- impose extra symmetries, e.g. R-parity $\sim$ matter parity
  $\rightarrow$ MSSM with 124 parameters Haber (hep-ph/9709450)

R-parity and matter parity

superfield formalism, e.g.

$$\boxed{Q = \widetilde{q} + q\,\theta + F_q\,\theta^2}$$

$q =$ quark

$\widetilde{q} =$ squark

$\theta =$ superspace variable

- R-parity (R_p) defined on component fields, e.g. q and $\widetilde{q}$

R_p	quarks	leptons	Higgs	squarks	sleptons	Higgsino
additive Z_2	0	0	0	1	1	1

- matter parity (M_p) defined on superfields, e.g. Q

M_p	(s)quarks	(s)leptons	Higgs(inos)
additive Z_2	1	1	0

R_p and M_P allow and forbid exactly the same terms in Lagrangian

Matter parity from $U(1)_{B-L}$

matter parity

- forbids (renormalizable) B and L violation
- introduced to stabilize proton
- violated by quantum gravity effects Krauss, Wilczek (1989)
- unless gauge origin, e.g. from breaking $U(1)_{B-L}$

	Q	U^c	D^c	L	E^c	H_u	H_d	ϕ
$U(1)_{B-L}$	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	1	0	0	$\pm\frac{2}{3}$
$3\times U(1)_{B-L}$	1	-1	-1	-3	3	0	0	± 2
M_p	1	1	1	1	1	0	0	0

$\rightarrow$ vev of field ϕ breaks $U(1)_{B-L}$ spontaneously down to M_p

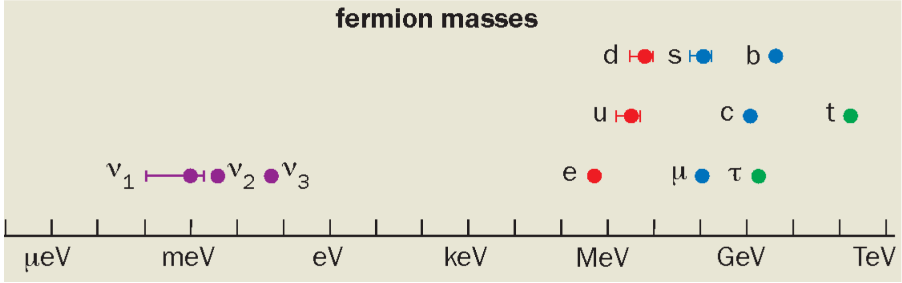
"Higgs"-type field
neutral under
SM gauge group

$U(1)_{\mathrm{FN}}$ family symmetry

Froggatt, Nielsen (1979)

Fermion masses

- fermion mass terms: $M_{ij}\,\overline{\psi}_i\,\psi_j$
- masses = eigenvalues of M



$$m_u : m_c : m_t \sim \lambda^8 : \lambda^4 : 1$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1$$

$$m_e : m_\mu : m_\tau \sim \lambda^{4\,\text{or}\,5} : \lambda^2 : 1 \qquad (\lambda \sim 0.22)$$

- neutrinos massless in Standard Model
- observation of neutrino oscillation $\rightarrow\; m_\nu \neq 0$
- three scenarios:

$$m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \sim \begin{cases} \lambda^x : \lambda : 1 & \text{(normal hierarchy)} \\ 1 : 1 : \lambda^x & \text{(inverted hierarchy)} \\ 1 : 1 : 1 & \text{(quasi degenerate)} \end{cases}$$

Froggatt-Nielsen mechanism

- Yukawa terms $\boxed{Y_{ij}\,\overline{\psi}_i\,\psi_j H}$ forbidden by $U(1)_{\mathrm{FN}}$
- family dependent charges
- introduce flavon field ϕ which allows effective terms $\boxed{c_{ij}\,\overline{\psi}_i\,\psi_j H\left(\frac{\phi}{\Lambda}\right)^{x_{ij}}}$
- flavon ϕ acquires a VEV $\rightarrow$ $Y_{ij} = c_{ij}\left(\frac{\langle\phi\rangle}{\Lambda}\right)^{x_{ij}}$

fields	Q_1	Q_2	Q_3	D_1^c	D_2^c	D_3^c	H_d	ϕ
$U(1)_{\mathrm{FN}}$	6	4	0	5	3	3	-3	-2

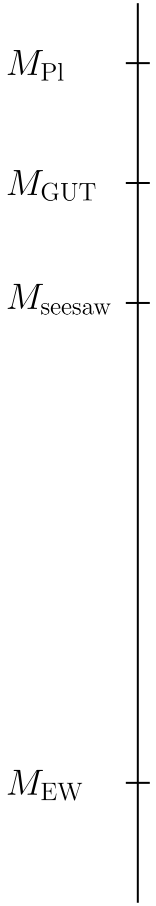
$$\frac{\langle\phi\rangle}{\Lambda} \sim \lambda$$

$$c_{ij}\,Q_i\,D_j^c\,H_d\left(\frac{\phi}{\Lambda}\right)^{x_{ij}} \rightarrow \quad c_{ij}\,Q_i\,D_j^c\,H_d\,\lambda^{x_{ij}} \rightarrow \quad Y_d \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}$$

- hierarchies arise from spontaneous breakdown of $U(1)_{\mathrm{FN}}$
- often residual discrete Z_N symmetry, e.g. matter parity
- $\mathcal{O}(1)$ coefficients c_{ij} not fixed

Scale of family symmetry breaking

- models of flavor are typically formulated at high energies
- separate family symmetry from EW symmetry breaking

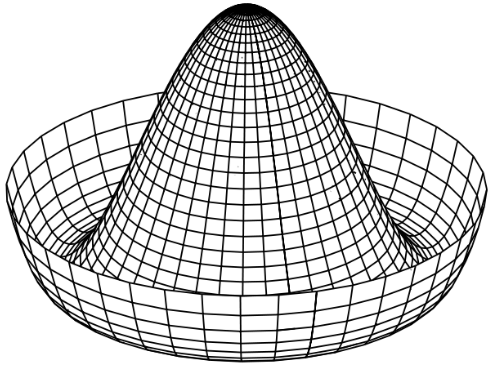


flavon VEV $\langle\phi\rangle$ breaks family symmetry

$\rightarrow$ Yukawas Y_{ij}

Higgs VEV $\langle H\rangle$ breaks electroweak symmetry

$\rightarrow$ fermion masses $M_{ij} = Y_{ij}\langle H\rangle$



Discrete symmetries from $U(1)_{\mathrm{FN}}$

- simple conditions to obtain a particular Z_N symmetry from $U(1)_{\mathrm{FN}}$
- in $U(1)_{\mathrm{FN}}$ charge normalization with $X_\phi = -1$

	$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$
M_p	integer $- \frac{1}{2}$	integer
P_6	integer $- \frac{1}{2}$	integer $\pm \frac{1}{3}$
B_3	integer	integer $\pm \frac{1}{3}$

Dreiner, Luhn, Murayama, Thormeier (hep-ph/0610026)

M_p lightest SUSY particle (LSP) is stable $\rightarrow$ dark matter candidate
allows for non-renormalizable operator $QQQL$ $\rightarrow$ proton decay

P_6 LSP is stable $\rightarrow$ dark matter candidate
forbids $QQQL$ $\rightarrow$ proton stabilized

B_3 LSP not stable $\rightarrow$ less missing E_T
forbids $QQQL$ $\rightarrow$ proton stabilized
renormalizable L violation $\rightarrow$ neutrino masses without seesaw

Discrete Z_4^R symmetry from $U(1)^R_{\mathrm{FN}}$

- superspace variable θ charged under R-symmetries

	Q	U^c	D^c	L	E^c	H_u	H_d	θ
Z_4^R	1	1	1	1	1	0	0	1

Lee et al.
(arXiv:1009.0905)

- solves the μ-problem by forbidding $\mu H_u H_d$
- forbids $QQQL$

- Z_4^R can be obtained from $U(1)^R_{\mathrm{FN}}$ if

$X_{H_d} - X_{L_1}$	$3X_{Q_1} + X_{L_1}$	X_θ	X_ϕ
integer $\pm \frac{1}{4}$	integer	integer $\pm \frac{1}{4}$	-1

Dreiner, Luhn,
Opferkuch
(in preparation)

Non-Abelian family symmetries

Fermion mixings

- mismatch of flavor (weak) and mass eigenstates

$$\Psi_{\text{flavour}} \;=\; V^\dagger\,\Psi_{\text{mass}}$$

- quark sector: V_L^u and V_L^d

$$U_{\text{CKM}} \;=\; V_L^u\,V_L^{d\,\dagger} \;\approx\; \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.22$$

- lepton sector: V_L^e and V_L^ν

$$U_{\text{PMNS}} \;=\; V_L^e\,V_L^{\nu\,\dagger} \;\approx\; \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

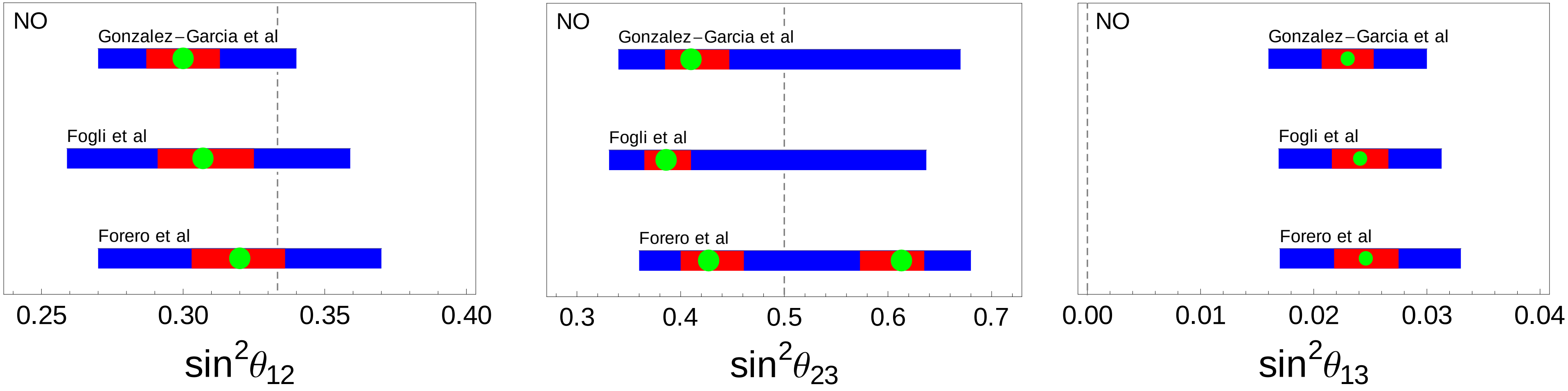
Gonzalez-Garcia et al. (2012)

mixing $\Longleftrightarrow$ each family knows of the existence of the others!

Tri-Bimaximal lepton mixing vs. global neutrino fits

$$U_{\mathrm{PMNS}} \approx U_{\mathrm{TB}} \equiv \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

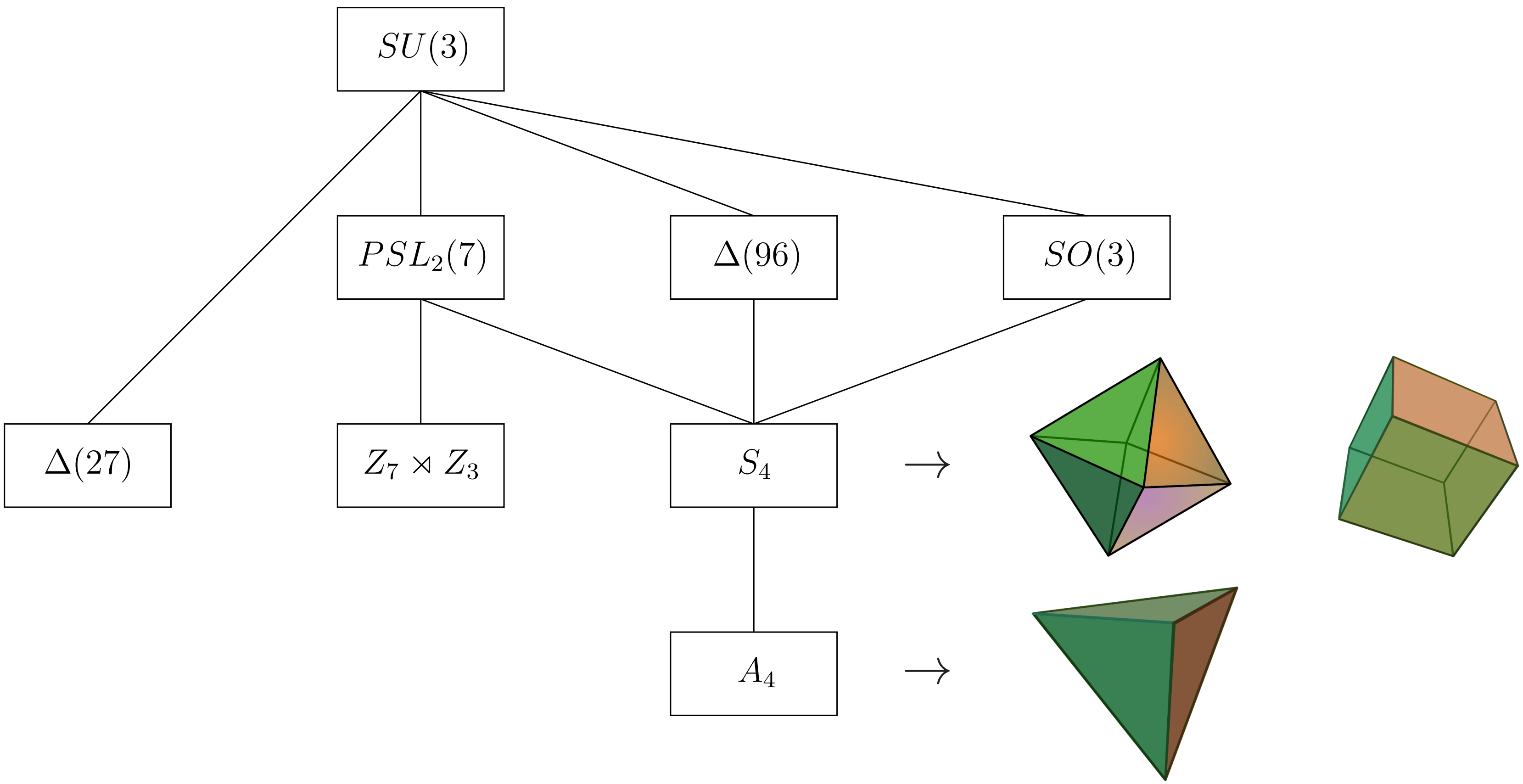
Harrison Perkins Scott



- TBmax mixing fits θ_{12} and θ_{23} very well – until recently also θ_{13}
- motivates family symmetries, e.g. A_4, S_4

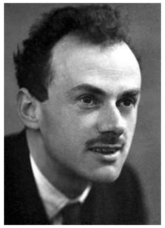
Candidates

- unify three families in multiplets of family symmetry
- underlying group should have two- or three-dimensional representations



Symmetries of the mass matrices (in flavor basis)

charged leptons $M_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$



Dirac

symmetric under diagonal phase transformation h

$$\boxed{M_\ell = h^T M_\ell h^*}$$

e.g. $h = \mathrm{diag}(1, e^{\frac{4\pi i}{3}}, e^{\frac{2\pi i}{3}})$

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neutrinos $M_\nu = U_{\mathrm{PMNS}}\,\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\,U_{\mathrm{PMNS}}^T$

Majorana

symmetry of M_ν depends on U_{PMNS}

$$\boxed{M_\nu = k^T M_\nu k}$$

$k = U^*_{\mathrm{PMNS}}\,\mathrm{diag}(+1, -1, -1)\,U^T_{\mathrm{PMNS}}$

four different k $\to$ generate $Z_2 \times Z_2$ symmetry group

Klein symmetry: $\mathcal{K} = \{1, k_1, k_2, k_3\}$

for $U_{\mathrm{PMNS}} = U_{\mathrm{TB}}$:

$$k_1 = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad k_2 = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad k_3 = k_1 k_2$$

Origin of the Klein symmetry $\mathcal{K}$

- direct models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \ \subset \ \text{family symmetry } \mathcal{G}}$
  - flavons ϕ are multiplets of $\mathcal{G}$
  - their VEVs $\langle\phi\rangle$ break $\mathcal{G}$ down to $\mathcal{K}$ in neutrino sector
  - for TBmax mixing (k_1, k_2, h) generate permutation group S_4
- indirect models
  - $\boxed{\text{Klein symmetry } \mathcal{K} \ \not\subset \ \text{family symmetry } \mathcal{G}}$
  - $\mathcal{G}$ responsible for generating particular flavon VEV configurations $\langle\phi\rangle$
  - for TBmax mixing – from e.g. $\Delta(27)$, $Z_7 \rtimes Z_3$

$$\langle\phi_1\rangle \propto \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_2\rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \langle\phi_3\rangle \propto \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\nu \quad \sim \quad \nu\,(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\,\nu\ H\,H$$

(exact) Tri-Bimaximal mixing is ruled out !

T2K [arXiv:1106.2822]

- $\theta_{13} \neq 0$ disfavored at $\sim 2.5\sigma$
- $5° \lesssim \theta_{13} \lesssim 18°$ at 90% C.L.

Double Chooz [arXiv:1207.6632]

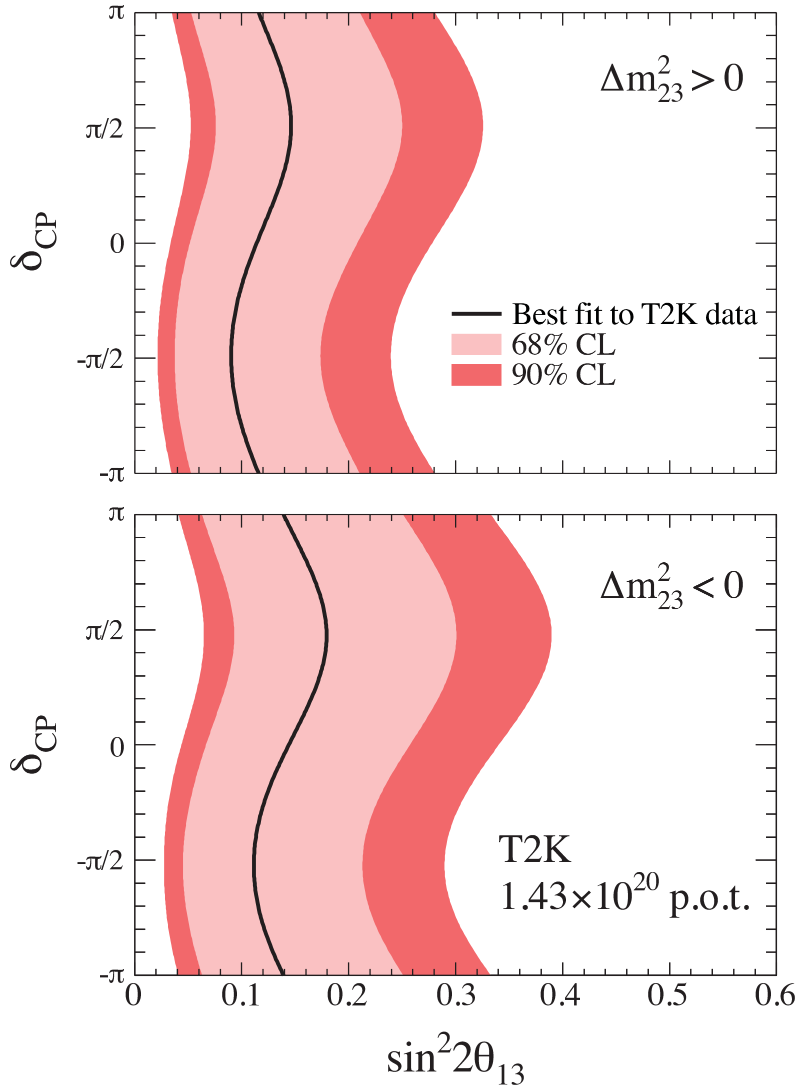
- $\theta_{13} \neq 0$ disfavored at $\sim 2.9\sigma$
- $6° \lesssim \theta_{13} \lesssim 12°$ at 90% C.L.

RENO [arXiv:1204.0626]

- $\theta_{13} \neq 0$ disfavored at $\sim 4.9\sigma$
- $8.0° \lesssim \theta_{13} \lesssim 11.4°$ at 90% C.L.

Daya Bay [arXiv:1210.6327]

- $\theta_{13} \neq 0$ disfavored at $\sim 7.7\sigma$
- $7.7° \lesssim \theta_{13} \lesssim 9.6°$ at 90% C.L.



Requires new model building strategies

<u>direct models</u>

TBmax plus corrections

other family symmetries
with non-standard $\mathcal{K}$

<u>indirect models</u>

TBmax plus corrections

non-standard flavon
VEV configurations

A direct model of leptons based on S_4

- discrete family symmetry with 24 elements
- irreducible S_4 representations: $\mathbf{1} \quad \mathbf{1'} \quad \mathbf{2} \quad \mathbf{3} \quad \mathbf{3'}$
- diagonal charged leptons
- in neutrino sector $\quad (L \sim N^c \sim \mathbf{3} \qquad H_u \sim \mathbf{1})$

$$\boxed{W_\nu \;\sim\; L N^c H_u + (\phi_{\mathbf{3'}} + \phi_{\mathbf{2}} + \phi_{\mathbf{1}}) N^c N^c + \frac{1}{M} \widetilde{\phi}_{\mathbf{1'}} \phi_{\mathbf{2}} N^c N^c}$$

S_4 irrep	k_1	k_2	VEV alignment
$\mathbf{3'}$	$\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{3'}} \rangle \propto \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{2}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\langle \phi_{\mathbf{2}} \rangle \propto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
$\mathbf{1}$	1	1	$\langle \phi_{\mathbf{1}} \rangle \propto 1$
$\mathbf{1'}$	1	-1	$\langle \widetilde{\phi}_{\mathbf{1'}} \rangle \propto 1$

$\longrightarrow$ TBmax Klein symmetry broken in k_2 (at higher order)

Resulting neutrino mixing

$$U_{\text{PMNS}} \approx \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \\ \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{2}}\alpha & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\alpha^* \end{pmatrix}$$

$\theta_{13} \approx 9^\circ$
requires
$|\alpha| \sim 0.2$

- second column of $U_{\text{PMNS}} \propto (1,1,1)^T$
- trimaximal mixing – due to k_1 symmetry

- TBmax breaking caused by only one free (complex) parameter
- affects θ_{13} and θ_{23} – but not θ_{12}
- get correlations between mixing parameters

$\Longrightarrow$ testable mixing sum rules

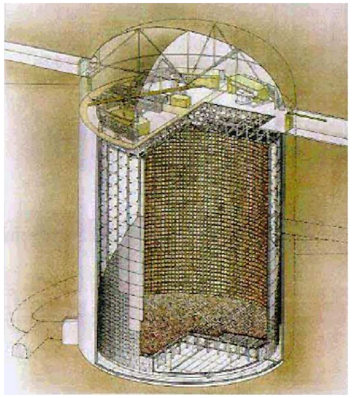
$$\theta_{23} \approx 45^\circ - \frac{1}{\sqrt{2}}\,\theta_{13}\cos\delta \qquad \theta_{12} \approx 35.3^\circ \text{ (TBmax value)}$$

Conclusion and outlook

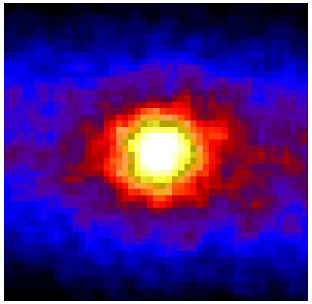
- discrete symmetries essential in supersymmetry (proton decay)
  - $Z_n^{(R)}$ symmetries from $U(1)_{\mathrm{FN}}^{(R)}$
  - R-parity violating symmetries might be interesting
- observed pattern in neutrino mixing
  - suggestive of non-Abelian (discrete) family symmetries
  - $\theta_{13} \approx 9^\circ$ from Daya Bay and RENO
  - deviations from tri-bimaximal mixing
  - testable mixing sum rules
- other aspects
  - ameliorate SUSY flavor problem
  - combine family and CP symmetry

Thank you

Neutrino oscillations: experimental milestones



- atmospheric neutrinos
  - · ν_μ / $\overline{\nu}_\mu$ disappear – Super-Kamiokande (1998)
- accelerator neutrinos
  - · ν_μ disappear – K2K (2002), MINOS (2006)
  - · ν_μ converted to ν_τ – OPERA (2010 & 2012)
  - · ν_μ converted to ν_e – T2K, MINOS (2011)



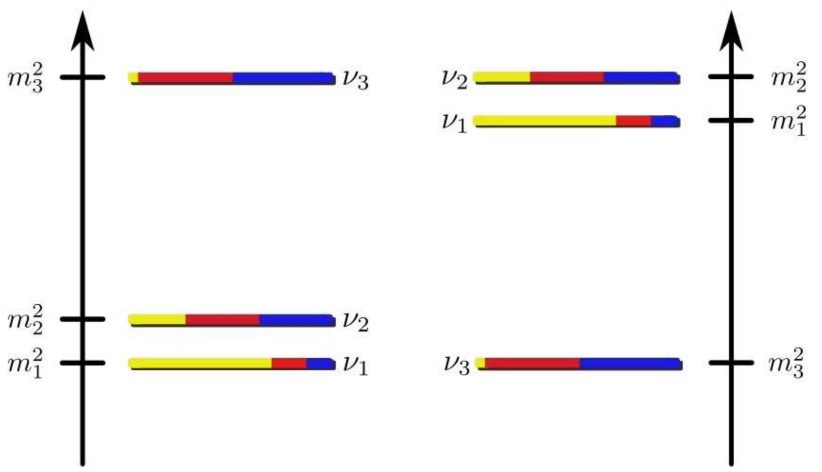
- solar neutrinos
  - · ν_e disappear – Chlorine (1998), Gallium (1999 - 2009), Super-Kamiokande (2002), Borexino (2008)
  - · ν_e converted to $(\nu_\mu + \nu_\tau)$ – SNO (2002)
- reactor neutrinos
  - · $\overline{\nu}_e$ disappear – Double Chooz (2011), Daya Bay, RENO (2012)
  - · $\overline{\nu}_e$ disappear – KamLAND (2002)

Three neutrino flavor mixing

(in diagonal charged lepton basis)

flavor PMNS mixing mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



atmospheric reactor + Dirac solar Majorana

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{\frac{\alpha_3}{2}} \end{pmatrix}$$

$\theta_{23} \approx 45^\circ$ $\theta_{13} \approx 9^\circ$ $\theta_{23} \approx 33^\circ$