On precision calculations in nonleptonic B-decays

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Outline

- Introduction to non-leptonic B decays
- QCD factorisation framework
- NNLO corrections in QCD factorisation
  - Tree amplitudes
  - Penguin amplitudes
  - The decay $B \to D\pi$
- Conclusion

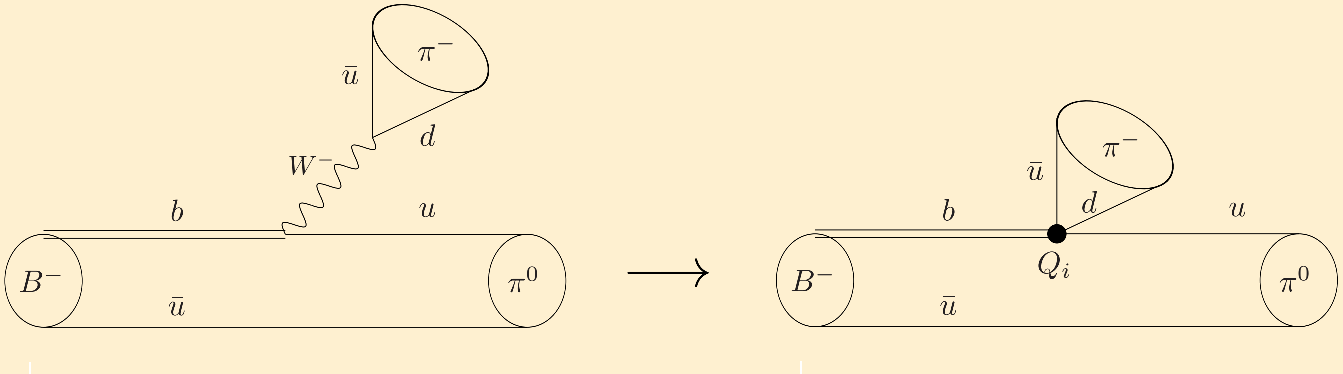
Introduction to non-leptonic B decays

- Non-leptonic B decays offer a rich and interesting phenomenology
  - Large data sets from B-factories, Tevatron, LHCb, future Super-flavour factory
  - $\mathcal{O}(100)$ final states
  - Numerous observables:
    - branching ratios
    - CP asymmetries
    - polarisations
    - Dalitz plot analyses
    - Combinations thereof
- Test of CKM mechanism (CP violation)
- Indirect search for New Physics
  - Not as sensitive as rare or radiative B decays, but large data sets
  - Smoking gun: $\Delta A_{\mathrm{CP}}(\pi K)$

Introduction to non-leptonic B decays

- Theoretical description complicated by purely hadronic initial and final state
- QCD effects from many different scales
- Theory approaches
  - (QCD improved) Factorisation
    - Disentangles long and short distances systematically
    - Problems with factorisation of power suppressed and annihilation contributions. Endpoint divergences
  - Flavour symmetries: Isospin, U-Spin ($d \leftrightarrow s$), V-Spin ($u \leftrightarrow s$), Flavour SU(3)
    - Only few a priori assumptions about scales needed
    - Implementation of symmetry breaking difficult
  - Dalitz plot analysis. "Fit to data"
    - Not a QCD theory prediction
    - Applied to 3-body decays

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark

- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

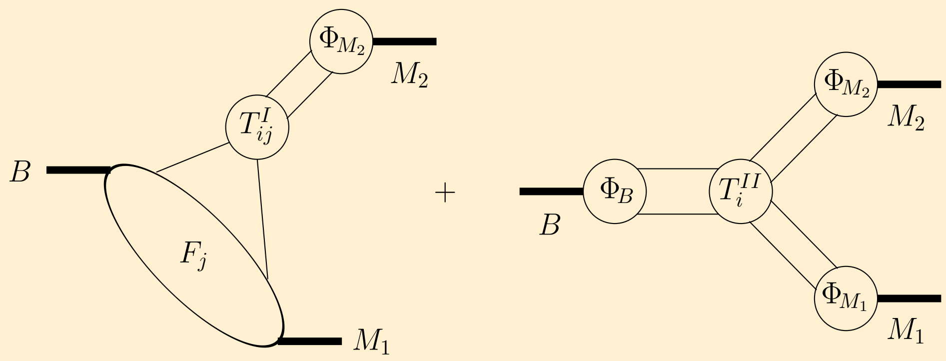
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2} m_b \, \bar{d}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

QCD factorisation



- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \;\, T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \;\, T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

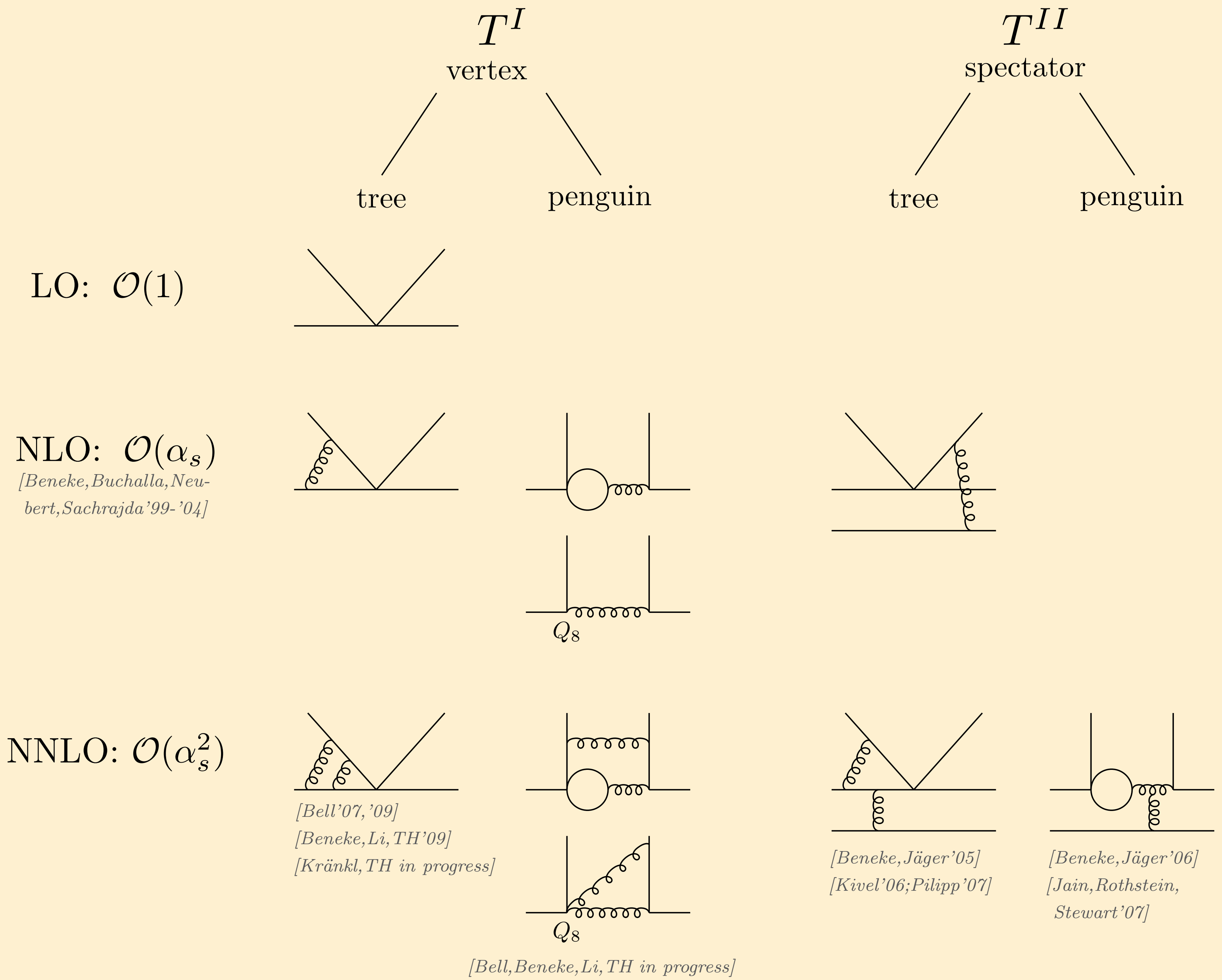
- $T^{I,II}$: Hard scattering kernels, perturbatively calculable

- $F_+ : B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

  } Universal. From Sum Rules, Lattice

- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

Anatomy of QCD factorisation



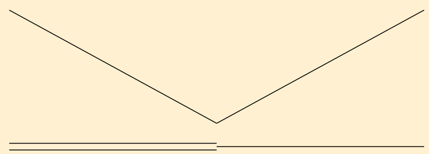
Classification of amplitudes

$$\begin{aligned}
\sqrt{2}\,\langle \pi^- \pi^0 | \mathcal{H}_{eff} | B^- \rangle &= A_{\pi\pi}\;\lambda_u \big[\alpha_1(\pi\pi) + \alpha_2(\pi\pi)\big] \\
\langle \pi^+ \pi^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle &= A_{\pi\pi}\;\big\{\lambda_u \big[\alpha_1(\pi\pi) + \alpha_4^u(\pi\pi)\big] + \lambda_c\,\alpha_4^c(\pi\pi)\big\} \\
-\,\langle \pi^0 \pi^0 | \mathcal{H}_{eff} | \bar{B}^0 \rangle &= A_{\pi\pi}\;\big\{\lambda_u \big[\alpha_2(\pi\pi) - \alpha_4^u(\pi\pi)\big] - \lambda_c\,\alpha_4^c(\pi\pi)\big\}
\end{aligned}$$

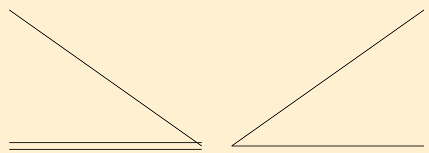
$$\begin{aligned}
\langle \pi^- \bar{K}^0 | \mathcal{H}_{eff} | B^- \rangle &= A_{\pi\bar{K}}\;\Big[\lambda_u^{(s)}\,\alpha_4^u + \lambda_c^{(s)}\,\alpha_4^c\Big] \\
\langle \pi^+ K^- | \mathcal{H}_{eff} | \bar{B}^0 \rangle &= A_{\pi\bar{K}}\Big[\lambda_u^{(s)}\,(\alpha_1 + \alpha_4^u) + \lambda_c^{(s)}\,\alpha_4^c\Big]
\end{aligned}$$

[Beneke,Neubert'03]

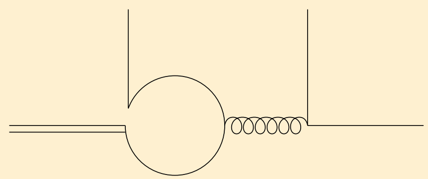
- α_1 : colour-allowed tree amplitude



- α_2 : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$: QCD penguin amplitudes



Motivation for NNLO

- NLO results

$$\alpha_1(\pi\pi) = 1.009 + [0.023 + 0.010\,i]_{\rm NLO} - \left[\tfrac{r_{\rm sp}}{0.445}\right]\left\{[0.014]_{\rm LOsp} + [0.008]_{\rm tw3}\right\} = 1.010 + 0.010i$$

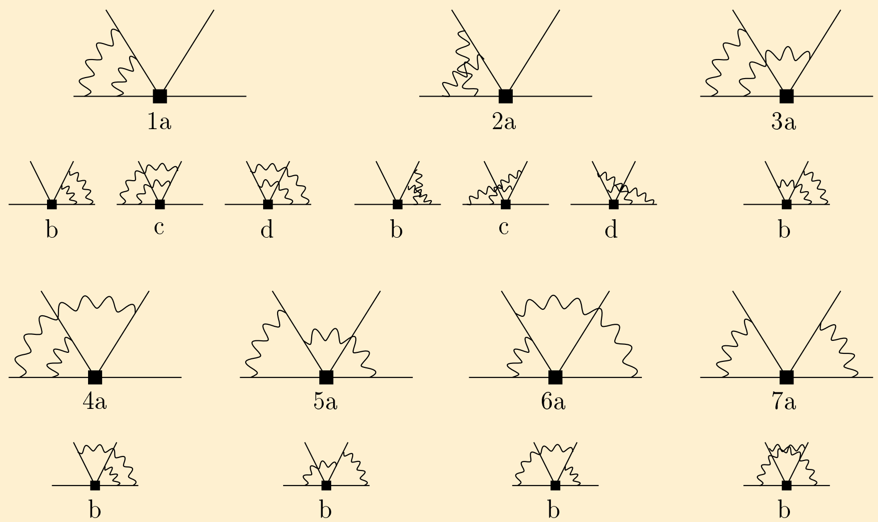
$$\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077\,i]_{\rm NLO} + \left[\tfrac{r_{\rm sp}}{0.445}\right]\left\{[0.114]_{\rm LOsp} + [0.067]_{\rm tw3}\right\} = 0.222 - 0.077i$$

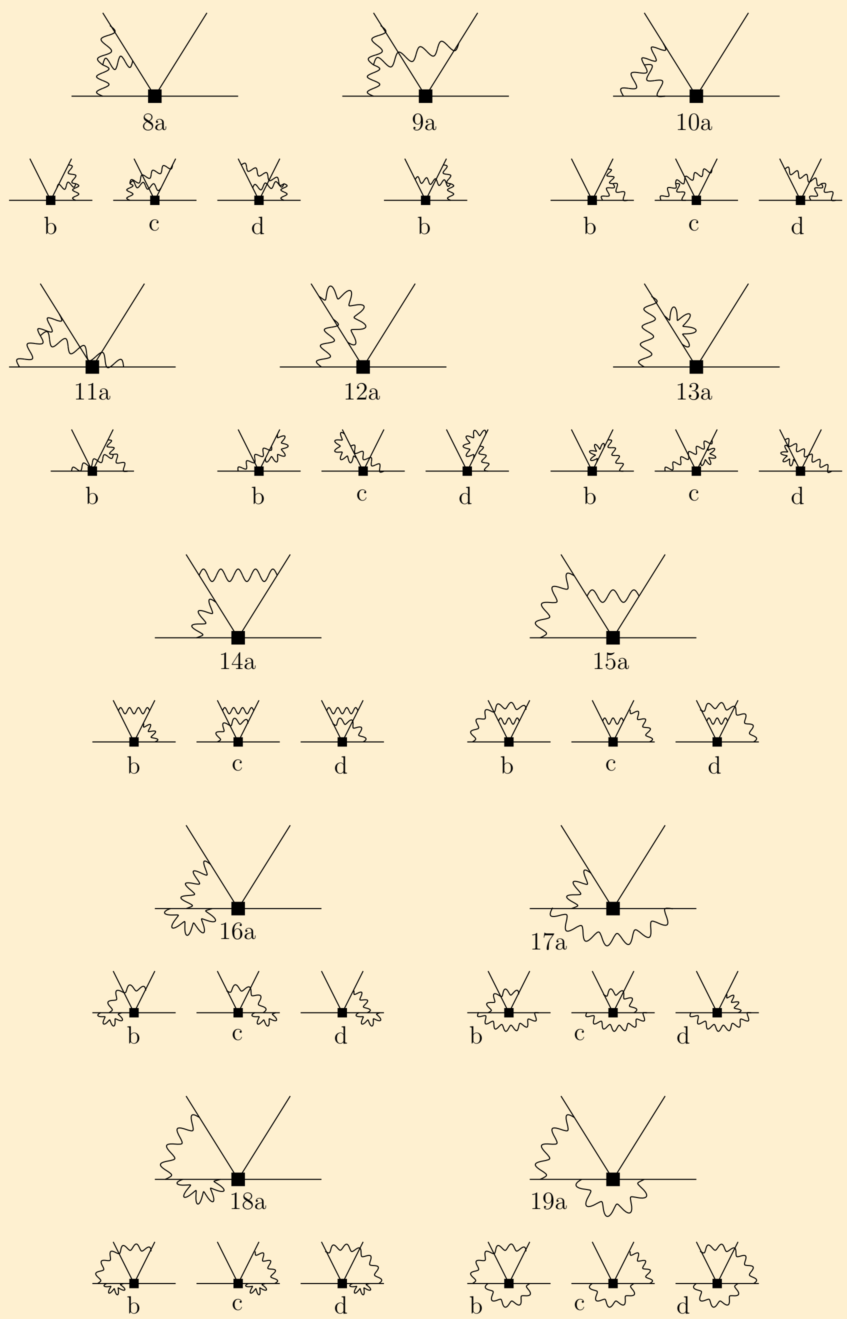
- Large cancellation in LO + NLO in α_2. Particularly sensitive to NNLO
- Direct CP asymmetries start at $\mathcal{O}(\alpha_s)$. NNLO is only first perturbative correction

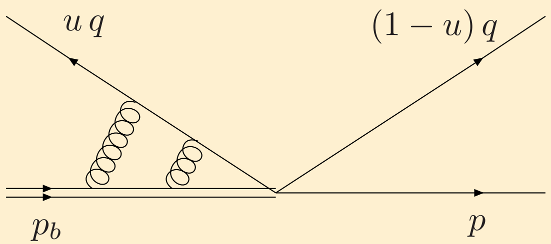
NLO QCDF	Experiment
$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.5 \pm 1.0) \times 10^{-6}$	$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.48^{+0.35}_{-0.34}) \times 10^{-6}$
$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.0 \pm 1.2) \times 10^{-6}$	$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.10 \pm 0.19) \times 10^{-6}$
$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (0.73 \pm 0.54) \times 10^{-6}$	$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.91^{+0.22}_{-0.23}) \times 10^{-6}$
[Beneke, Jäger'05]	*[PDG, HFAG]*

- Problems with "colour-suppressed" tree-dominated decays (e.g. $\bar{B}^0 \to \pi^0\pi^0$)
- Does NNLO QCDF tend toward the right direction?

Two-loop diagrams





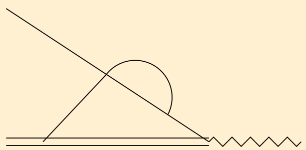


- Kinematics: $p_b^2 = m_b^2$
  $q^2 = 0$
  $p^2 = 0$ or $p^2 = m_c^2$

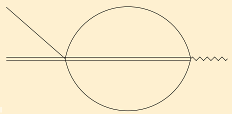
Computational methods

- Application of state-of-the-art multi-loop techniques
- Regularize UV and IR divergences dimensionally, $D = 4 - 2\epsilon$. Poles up to $1/\epsilon^4$.
- Integration-by-parts relations, Laporta algorithm

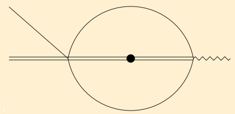
  [Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08; Studerus,von Manteuffel'10,'12]
- Obtain all integrals as a linear combination of master integrals



$$= \frac{(8-3D)(7uD-8D-24u+28)}{3(D-4)^2\, m_b^4\, u^3}$$



$$- \frac{2[u^2(D-4)+(16D-56)(1-u)]}{3(D-4)^2\, m_b^2\, u^3}$$



- Computation of ca. 40 master integrals
  - Hypergeometric functions *[see e.g. Maitre,TH'05,'07]*
  - Differential equations *[Kotikov'91; Remiddi'97]*
  - Mellin-Barnes representations *[Smirnov'99; Tausk'99; Czakon'05]*

Numerical Results at NNLO

- Obtain topological tree amplitudes completely analytically to NNLO

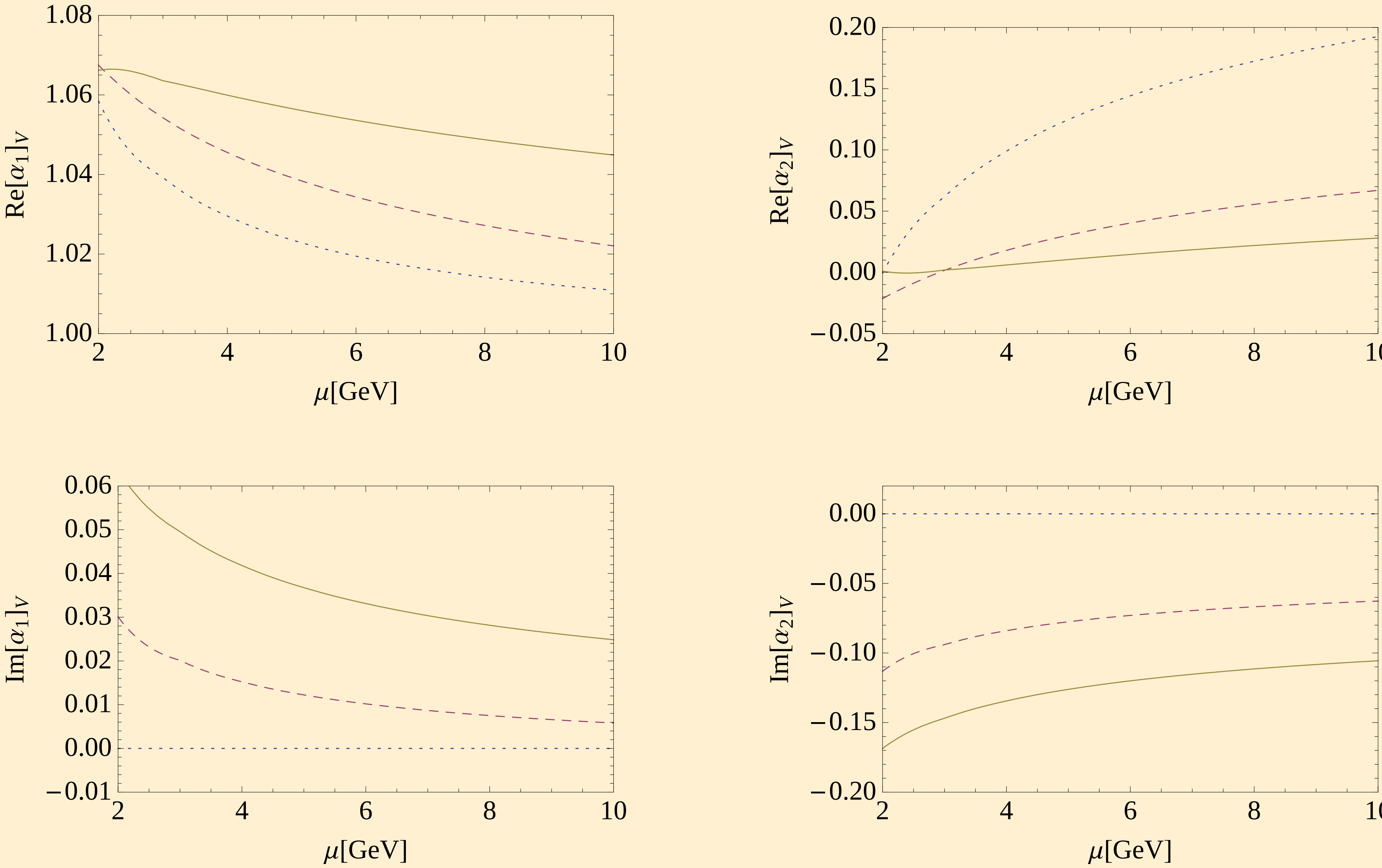
$$
\begin{aligned}
\alpha_1(\pi\pi) &= 1.009 + [0.023 + 0.010\,i]_{\rm NLO} + [0.026 + 0.028\,i]_{\rm NNLO} \\
&\quad - \left[\frac{r_{\rm sp}}{0.445}\right] \Big\{ [0.014]_{\rm LOsp} + [0.034 + 0.027\,i]_{\rm NLOsp} + [0.008]_{\rm tw3} \Big\} \\
&= 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})\,i
\end{aligned}
$$

$$
\begin{aligned}
\alpha_2(\pi\pi) &= 0.220 - [0.179 + 0.077\,i]_{\rm NLO} - [0.031 + 0.050\,i]_{\rm NNLO} \\
&\quad + \left[\frac{r_{\rm sp}}{0.445}\right] \Big\{ [0.114]_{\rm LOsp} + [0.049 + 0.051\,i]_{\rm NLOsp} + [0.067]_{\rm tw3} \Big\} \\
&= 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})\,i
\end{aligned}
$$

- NNLO corrections to vertex and spectator terms significant but tend to cancel!
- We find complete agreement (numerically) with G. Bell *[G. Bell'09]*

$$
r_{\rm sp} = \frac{9 f_{M_1} \hat{f}_B}{m_b \lambda_B f_+^{B\pi}(0)} \qquad\qquad \lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_B(\omega,\mu)
$$

Renormalization scale dependence



Factorisation test

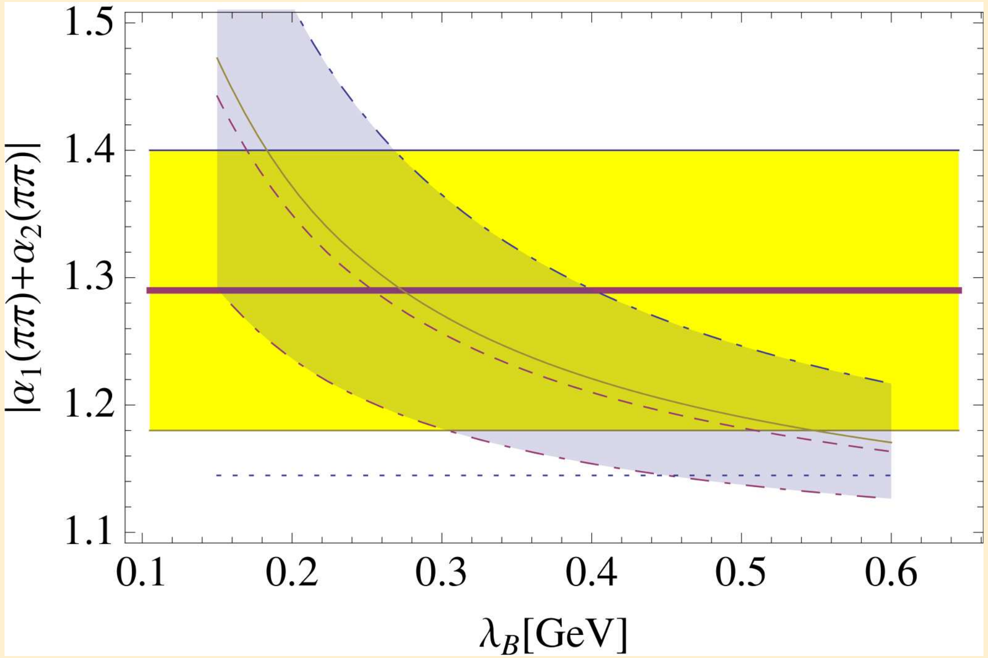
$$R \equiv \frac{\Gamma(B^- \to \pi^- \pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu})/dq^2\big|_{q^2=0}} = 3\pi^2 f_\pi^2 \, |V_{ud}|^2 \, |\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|^2$$

- From experimental data

  $$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\text{exp}} = 1.29 \pm 0.11$$

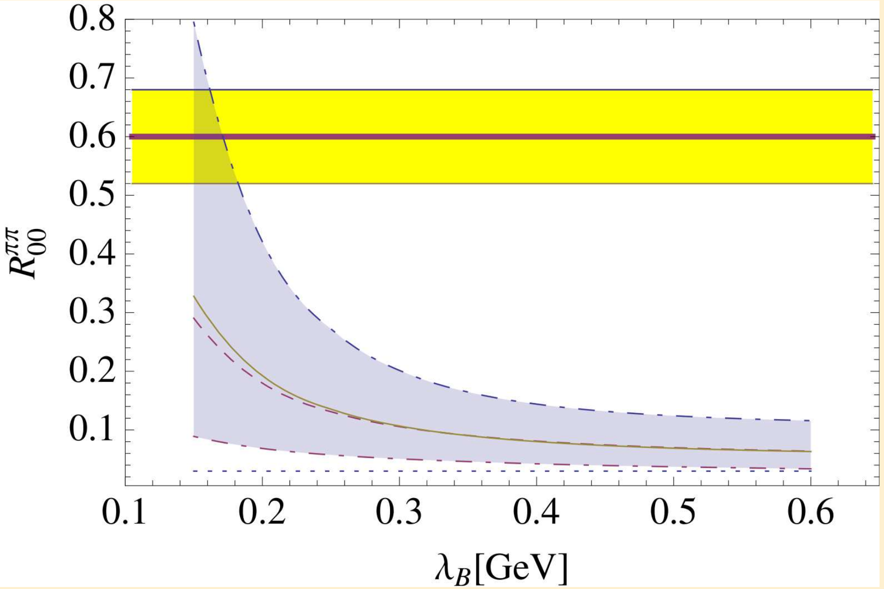
- Good agreement with theory supports QCDF approach

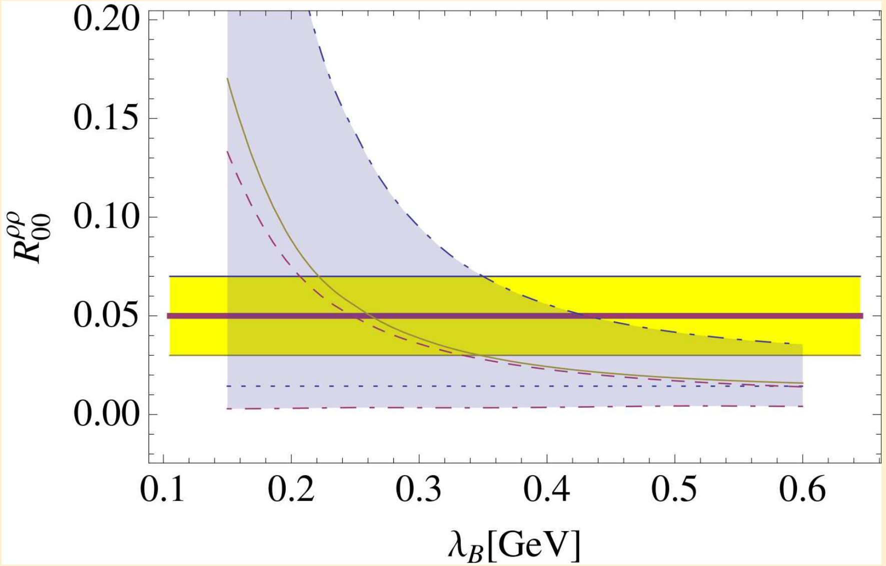
  $$|\alpha_1(\pi\pi) + \alpha_2(\pi\pi)|_{\text{th.}} = 1.24^{+0.16}_{-0.10}$$

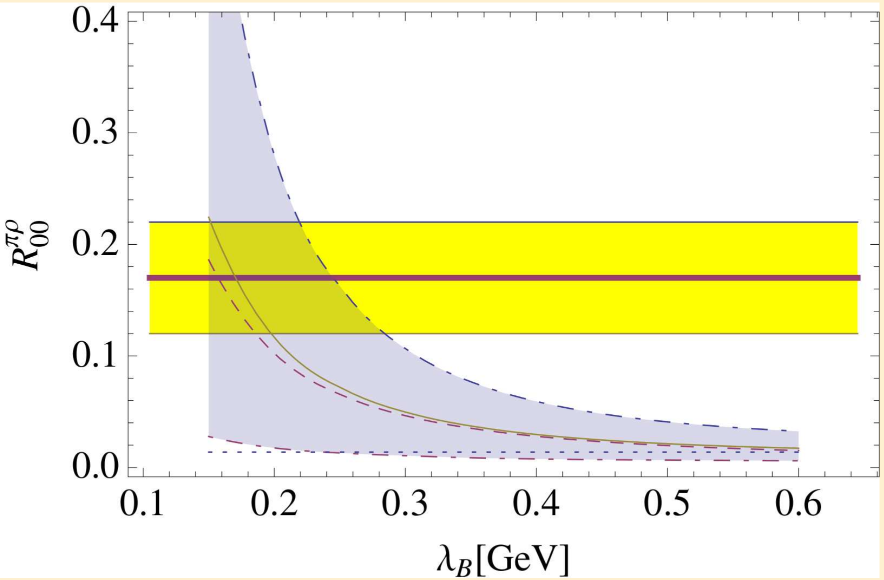


- Central exptl. value allows $\lambda_B \in [150, 400]$ MeV (on lower side of expectations).

Ratios involving colour-suppressed decays







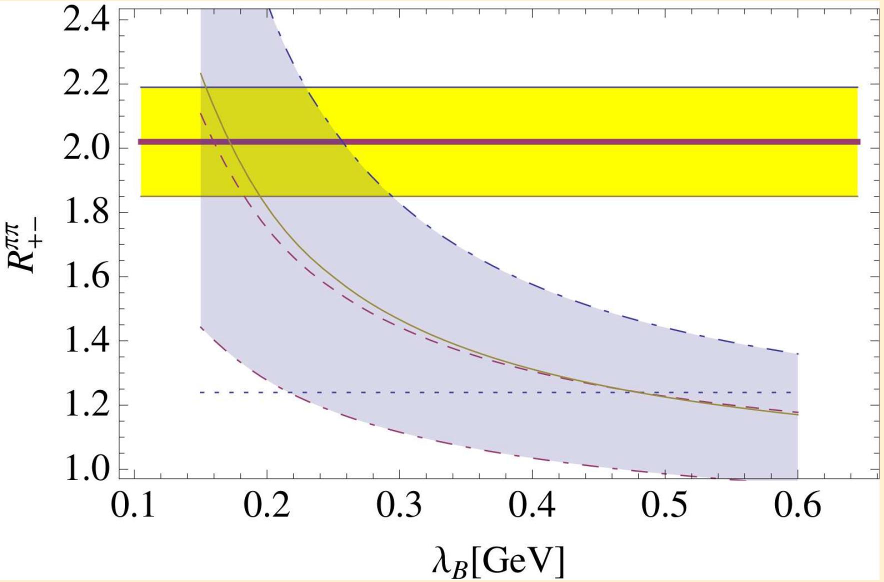
$$R_{00}^{\pi\pi} = 2\,\frac{\Gamma(B^0 \to \pi^0\pi^0)}{\Gamma(B^0 \to \pi^+\pi^-)}$$

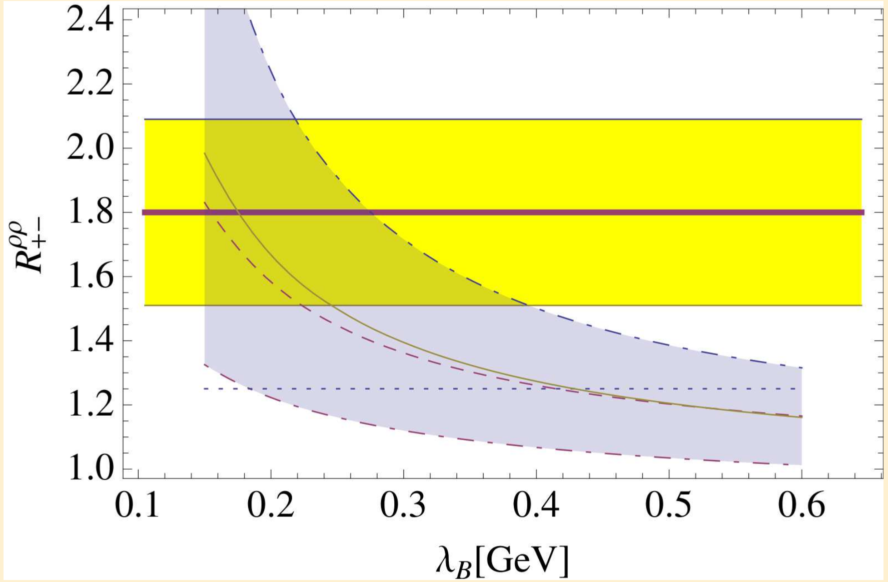
$$R_{00}^{\rho\rho} = 2\,\frac{\Gamma(B^0 \to \rho_L^0\rho_L^0)}{\Gamma(B^0 \to \rho_L^+\rho_L^-)}$$

$$R_{00}^{\pi\rho} = \frac{2\,\Gamma(B^0 \to \pi^0\rho^0)}{\Gamma(B^0 \to \pi^+\rho^-) + \Gamma(B^0 \to \pi^-\rho^+)}$$

Preference for small λ_B, i.e. strong spectator scattering

More ratios





$$R_{+-}^{\pi\pi} = 2\,\frac{\Gamma(B^\pm \to \pi^\pm \pi^0)}{\Gamma(B^0 \to \pi^+ \pi^-)}$$

$$R_{+-}^{\rho\rho} = 2\,\frac{\Gamma(B^\pm \to \rho_L^\pm \rho_L^0)}{\Gamma(B^0 \to \rho_L^+ \rho_L^-)}$$

Also here: Preference for small $\lambda_B \simeq 200$ MeV

Final numerical results

[See also Bell,Pilipp'09]

	Theory I	Theory II	Exp.
$B^- \to \pi^-\pi^0$	$5.43^{+0.06}_{-0.06}{}^{+1.45}_{-0.84}$	$5.82^{+0.07}_{-0.06}{}^{+1.42}_{-1.35}$	$5.48^{+0.35}_{-0.34}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86}_{-0.69}{}^{+1.22}_{-0.97}$	$5.70^{+0.70}_{-0.55}{}^{+1.16}_{-0.97}$	5.10 ± 0.19
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11}_{-0.08}{}^{+0.42}_{-0.17}$	$0.63^{+0.12}_{-0.10}{}^{+0.64}_{-0.42}$	$1.91^{+0.22}_{-0.23}$
$B^- \to \pi^-\rho^0$	$8.68^{+0.42}_{-0.41}{}^{+2.71}_{-1.56}$	$9.84^{+0.41}_{-0.40}{}^{+2.54}_{-2.52}$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90}_{-0.77}{}^{+2.18}_{-1.41}$	$12.13^{+0.85}_{-0.73}{}^{+2.23}_{-2.17}$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27}_{-0.19}{}^{+3.82}_{-3.50}$	$21.90^{+0.20}_{-0.12}{}^{+3.06}_{-3.55}$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04}_{-0.03}{}^{+1.11}_{-0.43}$	$1.49^{+0.07}_{-0.07}{}^{+1.77}_{-1.29}$	2.0 ± 0.5
$B^- \to \rho^-_L\rho^0_L$	$18.42^{+0.23}_{-0.21}{}^{+3.92}_{-2.55}$	$19.06^{+0.24}_{-0.22}{}^{+4.59}_{-4.22}$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\rho^-_L$	$25.98^{+0.85}_{-0.77}{}^{+2.93}_{-3.43}$	$20.66^{+0.68}_{-0.62}{}^{+2.99}_{-3.75}$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03}_{-0.03}{}^{+0.83}_{-0.36}$	$1.05^{+0.05}_{-0.04}{}^{+1.62}_{-1.04}$	$0.55^{+0.22}_{-0.24}$
$R^{\pi\pi}_{+-}$	$1.38^{+0.12}_{-0.13}{}^{+0.53}_{-0.32}$	$1.91^{+0.18}_{-0.20}{}^{+0.72}_{-0.64}$	$1.99^{+0.15}_{-0.14}$
$R^{\pi\pi}_{00}$	$0.09^{+0.03}_{-0.02}{}^{+0.12}_{-0.04}$	$0.22^{+0.06}_{-0.05}{}^{+0.28}_{-0.16}$	0.75 ± 0.09
$R^{\rho\rho}_{+-}$	$1.32^{+0.02}_{-0.03}{}^{+0.44}_{-0.27}$	$1.72^{+0.03}_{-0.03}{}^{+0.64}_{-0.53}$	$1.78^{+0.27}_{-0.28}$
$R^{\rho\rho}_{00}$	$0.03^{+0.00}_{-0.00}{}^{+0.07}_{-0.03}$	$0.10^{+0.01}_{-0.01}{}^{+0.19}_{-0.11}$	0.05 ± 0.02
$R^{\pi\rho}_{00}$	$0.04^{+0.00}_{-0.00}{}^{+0.09}_{-0.03}$	$0.14^{+0.01}_{-0.01}{}^{+0.20}_{-0.13}$	0.17 ± 0.05

- Theory II: With lower λ_B and form factors. Our preferred scenario.

Penguin amplitudes at two loops

[Work in progress, in collaboration with G. Bell, M. Beneke, X.-Q. Li]

- Penguin amplitudes to NLO

$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? \; + \; ?? \; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\tfrac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002} + (-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [?? \; + \; ?? \; i]_{\mathcal{O}(\alpha_s^2)}$$

$$+ \left[\tfrac{r_{\rm sp}}{0.485}\right] \left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.028^{+0.005}_{-0.003} + (-0.006^{+0.003}_{-0.002})i$$

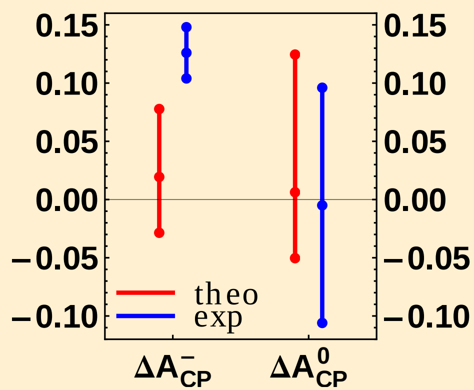
- Interesting quantity

$$\Delta A_{\rm CP}(\pi K) = A_{\rm CP}(B^- \to \pi^0 K^-) - A_{\rm CP}(\bar{B}^0 \to \pi^+ K^-)$$

$\Delta A_{\rm CP}(\pi K) = (12.6 \pm 2.2)\,\%$ (exp.) *[HFAG'12]*

$\Delta A_{\rm CP}(\pi K) = (1.9^{+5.8}_{-4.8})\,\%$ (NLO QCDF) *[Hofer,Vernazza'12]*

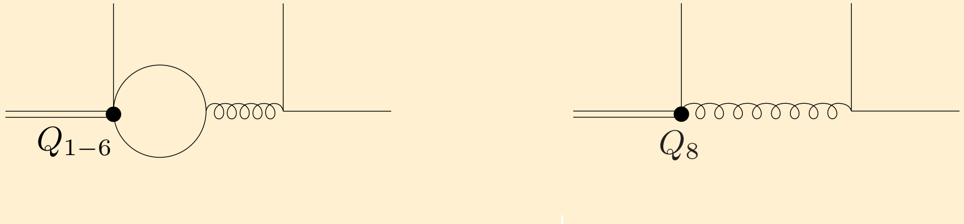
Tension at the 2.2σ level



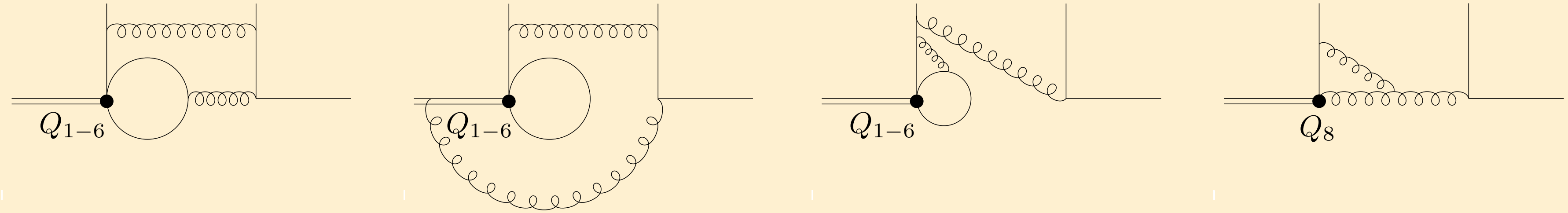
Penguin amplitudes at two loops

[Work in progress, in collaboration with G. Bell, M. Beneke, X.-Q. Li]

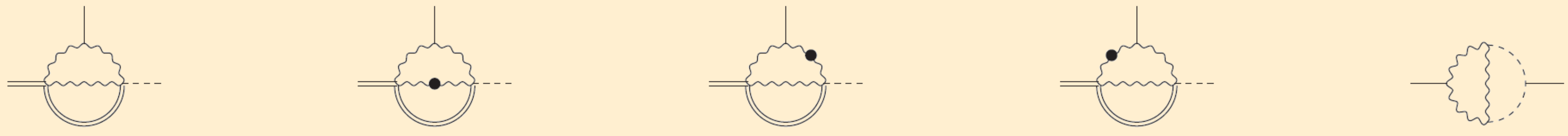
- NLO:



- $\mathcal{O}(70)$ diagrams at NNLO.



- We find 22 new master integrals for the penguin amplitudes



  - Double: m_b^2 , wavy: m_c^2 , solid: $u\, m_b^2$, dashed: 0 .
  - Genuine two-loop, two-scale problem: u , m_c^2/m_b^2

The decays $B \to D^{(*)}\,\pi\,/\,\rho$

- Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering power suppressed
- Applications
  - Ratios of decay widths

$$\frac{\Gamma(\bar{B}_d \to D^+\pi^-)}{\Gamma(\bar{B}_d \to D^{*+}\pi^-)} = \frac{(m_B^2 - m_D^2)^2|\vec{q}|_{D\pi}}{4m_B^2|\vec{q}|^3_{D^*\pi}} \left(\frac{F_0(m_\pi^2)}{A_0(m_\pi^2)}\right)^2 \left|\frac{a_1(D\pi)}{a_1(D^*\pi)}\right|^2$$

$$\frac{\Gamma(\bar{B}_d \to D^+\rho^-)}{\Gamma(\bar{B}_d \to D^+\pi^-)} = \frac{4m_B^2|\vec{q}|^3_{D\rho}}{(m_B^2 - m_D^2)^2|\vec{q}|_{D\pi}} \frac{f_\rho^2}{f_\pi^2} \left(\frac{F_+(m_\rho^2)}{F_0(m_\pi^2)}\right)^2 \left|\frac{a_1(D\rho)}{a_1(D\pi)}\right|^2$$

  - Test of factorisation

$$\frac{\Gamma(\bar{B}_d \to D^{(*)+}\pi^-)}{d\Gamma(\bar{B}_d \to D^{(*)+}l^-\bar{\nu})/dq^2\big|_{q^2=m_\pi^2}} = 6\pi^2|V_{ud}|^2\; f_\pi^2\; |a_1(D^{(*)}\pi)|^2$$

  - Angular analysis in case of $D^*\rho$
- NLO correction small: Colour suppression, small Wilson Coefficient
- Dirac and Laporta reduction and identification of master integrals complete ✓

$\longrightarrow$ See talk by S. Kränkl, T19.7

Conclusion

- Field of nonleptonic B decays has reached the era of precision physics
- The colour-allowed and colour-supressed tree amplitudes have been computed completely analytically to NNLO
- The NNLO corrections are small. Cancellation between vertex and spectator term
- QCD factorisation describes data on tree-dominated decays well.
  Exceptions are observables with $\pi^0\pi^0$ final state
- To further improve precision also need refined nonperturbative input from Sum Rules and Lattice QCD
- Work in progress:
  - Two-loop penguin amplitudes, CP asymmetries at NLO
  - $B \to D\pi$

Backup slides

Some definitions

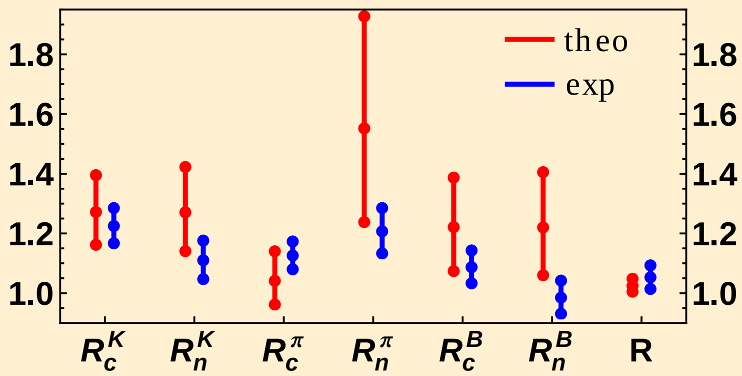
$$A_{\pi\pi} = i\,\frac{G_F}{\sqrt{2}}\,m_B^2 F_+^{B\to\pi}(0) f_\pi$$

$$r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)}$$

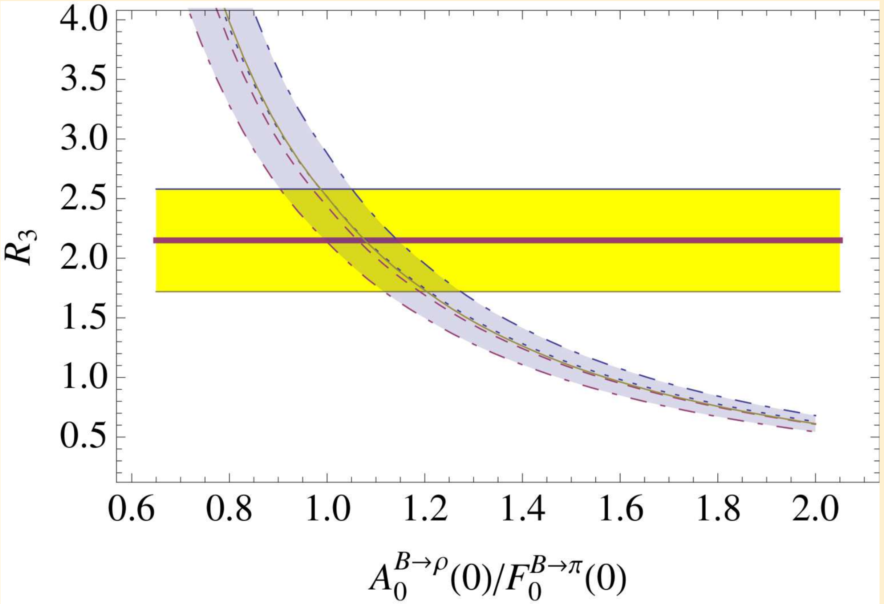
$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)$$

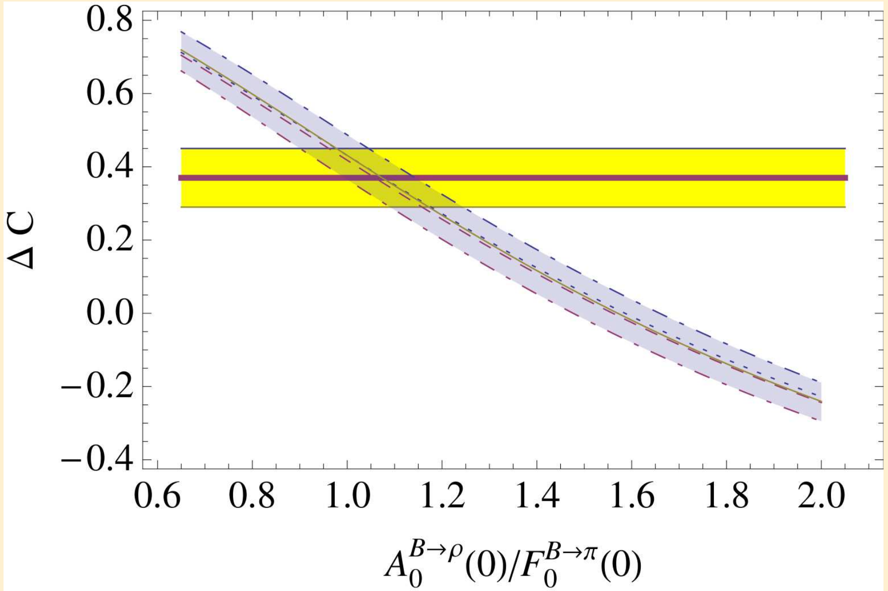
$$\Delta A_{\rm CP}^{-}(\pi K) = A_{\rm CP}(B^- \to \pi^0\,K^-) - A_{\rm CP}(\bar{B}^0 \to \pi^+\,K^-) = \Delta A_{\rm CP}(\pi K)$$

$$\Delta A_{\rm CP}^{0}(\pi K) = A_{\rm CP}(B^- \to \pi^-\,\bar{K}^0) - A_{\rm CP}(\bar{B}^0 \to \pi^0\,\bar{K}^0)$$



Dependence on form factors





$$R_3 = \frac{\Gamma(\bar{B}^0 \to \pi^+\rho^-)}{\Gamma(\bar{B}^0 \to \pi^-\rho^+)}$$

$$\Delta C = \frac{1}{2}\left[C(\pi^-\rho^+) - C(\pi^+\rho^-)\right]$$

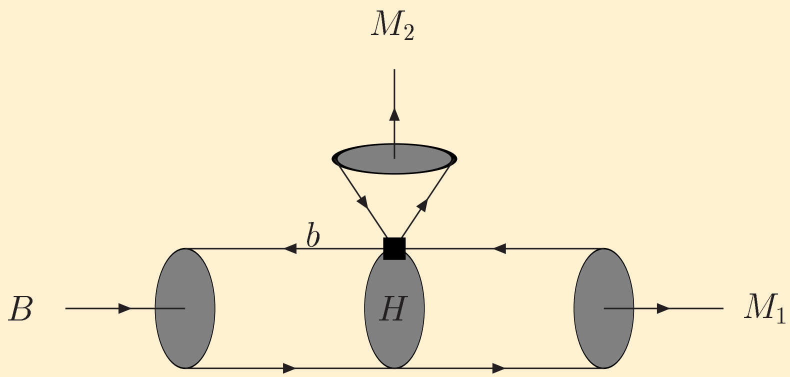
- Default value
  - $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) = 1.2$
- Agreement excellent for $A_0^{B\to\rho}(0)/F_0^{B\to\pi}(0) \in [1.0, 1.2]$

More on theory approaches to nonleptonic B-decays

- Perturbative QCD (PQCD) approach based on k_T-factorisation [see e.g. Keum,Li,Sanda'01]
  - Factorises amplitudes according to

$$A(B \to M_1 M_2) = \phi_B \otimes H \otimes J \otimes S \otimes \phi_{M_1} \otimes \phi_{M_2}$$

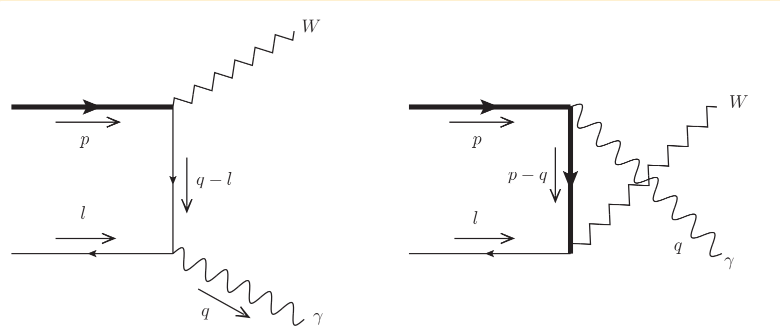
  - Generates larger strong phases. Avoids endpoint divergences.
  - However: Organises amplitude differently
  - Introduces additional infrared prescriptions, e.g. exponentiation of Sudakov logarithms, phenomenological model for transverse momentum effects
  - Discussion of theoretical uncertainties difficult, since no complete NLO ($\mathcal{O}(\alpha_s^2)$) analysis available
  - Independent information on hadronic input functions not available



λ_B from $B \to \gamma \ell \nu$

[Beneke,Rohrwild'11]

- First inverse moment of B-meson distribution amplitude: $\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega,\mu)$

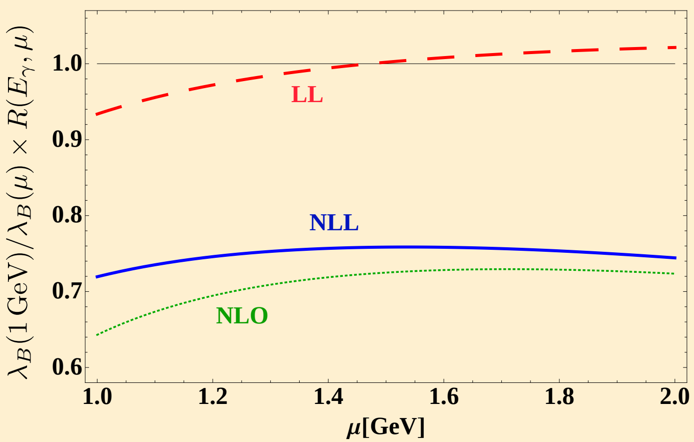


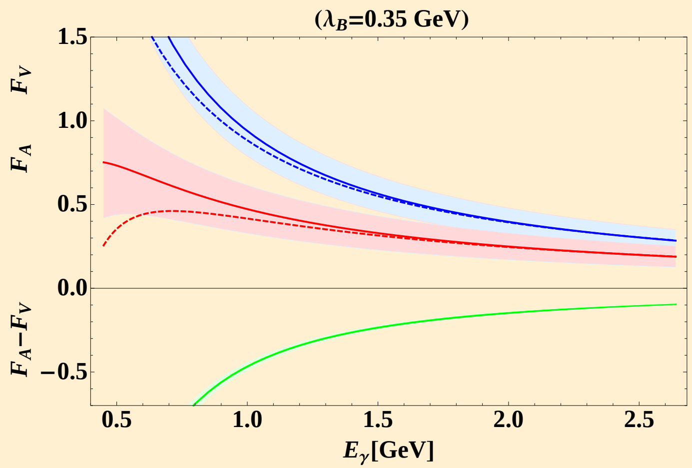
$$\frac{d^2\Gamma}{dE_\gamma\, dE_\ell} = \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{16\pi^2} m_B^3 (1-x_\gamma) \times \left[(1-x_\nu)^2 (F_A+F_V)^2 + (1-x_\ell)^2 (F_A-F_V)^2\right]$$

- Hard, energetic photon: $E_\gamma \lesssim m_B/2$
- Include NLL radiative and $1/m_b$ power corrections

$$F_V(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) + \frac{Q_b m_B f_B}{2E_\gamma m_b} + \frac{Q_u m_B f_B}{(2E_\gamma)^2}\right]$$

$$F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) - \frac{Q_b m_B f_B}{2E_\gamma m_b} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} + \frac{Q_\ell f_B}{E_\gamma}\right]$$





λ_B from $B \to \gamma\,\ell\,\nu$

[Beneke,Rohrwild'11; Braun,Khodjamirian'12]

- Soft overlap contribution to $\xi(E_\gamma)$ recently calculated
- Experimental upper bound yields lower bound on λ_B
- Power and NLL radiative corrections are significant and lower theory prediction
- This reduces the lower bound on λ_B to $\lambda_B \gtrsim 350\,\mathrm{MeV}$
- Small value for λ_B preferred by QCDF
- But: Need more data with large E_γ