

## SOE 19: Economic Models and Evolutionary Game Theory II (with BP, DY)

Time: Thursday 14:00–15:00

Location: GÖR 226

SOE 19.1 Thu 14:00 GÖR 226

**learning, evolution and population dynamics** — JUERGEN JOST and •WEI LI — Max-Planck-Institute for Mathematics in the Sciences

We study a complementarity game as a systematic tool for the investigation of the interplay between individual optimization and population effects and for the comparison of different strategy and learning schemes. The game randomly pairs players from opposite populations. It is symmetric at the individual level, but has many equilibria that are more or less favorable to the members of the two populations. Which of these equilibria is then attained is decided by the dynamics at the population level. Players play repeatedly, but in each round with a new opponent. They can learn from their previous encounters and translate this into their actions in the present round on the basis of strategic schemes. The schemes can be quite simple, or very elaborate. We can then break the symmetry in the game and give the members of the two populations access to different strategy spaces. Typically, simpler strategy types have an advantage because they tend to go more quickly toward a favorable equilibrium which, once reached, the other population is forced to accept. Also, populations with bolder individuals that may not fare so well at the level of individual performance may obtain an advantage toward ones with more timid players. By checking the effects of parameters such as the generation length or the mutation rate, we are able to compare the relative contributions of individual learning and evolutionary adaptations.

SOE 19.2 Thu 14:15 GÖR 226

**When does stochastic learning in game theory fixate ?** — JOHN REALPE-GOMEZ<sup>1</sup>, BARTOSZ SZCZESNY<sup>2</sup>, LUCA DALL'ASTA<sup>3</sup>, and •TOBIAS GALLA<sup>4</sup> — <sup>1</sup>Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy — <sup>2</sup>University of Leeds, Department of Applied Mathematics, School of Mathematics, Leeds LS2 9JT, UK — <sup>3</sup>The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy — <sup>4</sup>University of Manchester, School of Physics and Astronomy, Manchester M13 9PL, UK

Evolutionary dynamics in finite populations is known to fixate eventually in the absence of mutation. We here show that a similar phenomenon can occur in stochastic learning of a fixed set of players interacting repeatedly in a given game. We study in detail the mechanisms behind these absorption phenomena, in particular we present analytical predictions for the resulting fixation times and provide a detailed comparison with fixation in evolutionary dynamics. Specific examples are discussed, including simple two-player games, but also multi-player games defined on networks, resulting in more complicated interaction structures. In the final part of the talk I will discuss an imitation dynamics leading to absorption at fixed points in the interior of strategy space, a phenomenon not usually observed in standard models of evolutionary dynamics.

SOE 19.3 Thu 14:30 GÖR 226

**How small are small mutation rates ?** — BIN WU<sup>1,2</sup>, •CHAITANYA GOKHALE<sup>1</sup>, and ARNE TRAUlsen<sup>1</sup> — <sup>1</sup>Research Group

for Evolutionary Theory, Max-Planck-Institute for Evolutionary Biology, August-Thienemann-Str. 2, 24306 Plön, Germany — <sup>2</sup>Center for Systems and Control, State Key Laboratory for Turbulence and Complex Systems, College of Engineering, Peking University, Beijing, China

In recent years numerous analytical advances have been made in the field of evolutionary game theory. Some of them consider processes in which strategies can mutate between each other. Often the assumption of small mutation rates is made to keep the analysis tractable [1,2,3]. For small mutation rates the population is monomorphic most of the time. Occasionally a mutant arises. It can either reach fixation or go extinct. The evolutionary dynamics of the process under small mutation rates can be approximated by an embedded Markov chain on the pure states. Previously it was shown that in the limit of mutation rates going to zero the embedded Markov chain is a good approximation [4]. Here we derive an upper limit until where the approximation holds good. For a coexistence game it is *necessary* that the mutation rate  $\mu$  is less than  $N^{-1/2} \exp[-N]$  and for *all other* games, it is *sufficient* if the mutation rate is smaller than  $(N \ln N)^{-1}$ . Our results hold for a wide class of imitation processes under arbitrary selection intensity.

References: [1] Hauert C *et al.*, Science 316, 2007. [2] Van Segbroeck S *et al.*, Phys Rev Lett 102:058,105, 2009. [3] Sigmund K *et al.*, Nature 466, 2010. [4] Fudenberg D, Imhof LA, J Econ Theory 131, 2006.

SOE 19.4 Thu 14:45 GÖR 226

**Universality of weak selection** — BIN WU<sup>1,2</sup>, •PHILIPP M. ALTROCK<sup>1</sup>, LONG WANG<sup>2</sup>, and ARNE TRAUlsen<sup>1</sup> — <sup>1</sup>Max-Planck-Institute for Evolutionary Biology, Plön — <sup>2</sup>College of Engineering, Peking University, Beijing

Weak selection, which means a phenotype is slightly advantageous over another, is an important limiting case in evolutionary biology. Recently it has been introduced into evolutionary game theory. In evolutionary game dynamics, the probability to be imitated or to reproduce depends on the performance in a game. The influence of the game on the stochastic dynamics in finite populations is governed by the intensity of selection. In many models of both unstructured and structured populations, a key assumption allowing analytical calculations is weak selection, which means that all individuals perform approximately equally well. In the weak selection limit many different microscopic evolutionary models have the same or similar properties. How universal is weak selection for those microscopic evolutionary processes? We answer this question by investigating the fixation probability and the average fixation time not only up to linear, but also up to higher orders in selection intensity. We find universal higher order expansions, which allow a rescaling of the selection intensity. With this, we can identify specific models which violate (linear) weak selection results, such as the one-third rule of coordination games in finite but large populations.

[1] Wu, Altrock, Wang, and Traulsen, Physical Review E 82, 046106 (2010).