BP 18: Anomalous Transport II (joint BP, DY)

Time: Wednesday 11:15–13:15 Location: H38

BP 18.1 Wed 11:15 H38

Anomalous lateral diffusion in a layered medium — •Eugene B. Postnikov[1] and Igor M. Sokolov[2] — [1]Staatliche Universität Kursk, Russland — [2]Institut für Physik Humboldt - Universität zu Berlin, Deutschland

We consider the marker's diffusion in a layered medium, with the lateral diffusion coefficient being the function y-coordinate, i.e. the problem described by the diffusion equation for the marker density $u(x, y, t)$

$$\partial_t u = D_x(y)\partial_{xx}u + D_y\partial_{yy}u$$

with anisotropic diffusion coefficient $\hat{D}$. We show that the mean density averaged over the height, $U(x, t)$, follows the Bachelor's one-dimensional diffusion equation with time-dependent diffusion coefficient

$$\partial_t U = D_x(t)\partial_{xx}U$$

and obtain the expression of $D_x(t)$. As an example, we discuss the exact analytical solution in the case of a parabolic distribution $D_y \sim y^2$, leading to the anomalous (superdiffusive) behavior of a mean-square displacement $< x^2 >\propto t^{3/2}$. This result is confirmed by the numerical solution.

The approach is applied for the continual description of experimental results on inhomogeneous molecular diffusion in layered structures of thin liquid films deposited on solid surfaces [J. Schuster, F. Cichos, C. von Borzcyskowski. Eur. Polym. J. **40** (2004) 993].

BP 18.2 Wed 11:30 H38

From Anomalous Deterministic Diffusion to the Continuous-Time Random Walk — •Markus Niemann and Holger Kantz — Max-Planck-Institut für Physik komplexer Systeme, Dresden

There are several stochastic models describing anomalous diffusion. The most common are the fractional Brownian motion and the Continuous-Time Random Walk (CTRW). The question arises how to choose the correct model for a given process. Our approach is to look for deterministic foundations of these models. We present a method how to derive a CTRW as asymptotic description of a deterministic diffusion process.

We have introduced a diagrammatic method to determine the joint probability distributions of CTRWs. This method is extended to allow couplings between steps. These couplings may arise from deterministic maps, thereby allowing a unified treatment of stochastic and deterministic systems. Often, these processes converge in the scaling limit to a CTRW without coupling between steps. We apply the theory to a diffusion process driven by a deterministic map of Manneville-Pomeau type. Depending on the parameter, one gets a transition from an uncoupled to a coupled CTRW and a transition from sub- to superdiffusion. These findings are well supported by numerical simulations.

BP 18.3 Wed 11:45 H38

Random walks on d-dimensional Sierpinski gaskets: Asymptotics, DSI, and Puzzles — •Sebastian Weber[1], Joseph Klafter[2,1], and Alexander Blumen[3] — [1]Freiburg Institute For Advanced Studies (FRIAS), University of Freiburg, Germany — [2]School of Chemistry, Tel Aviv University, Israel — [3]Theoretical Polymer Physics, University of Freiburg, Germany

We study the effect of the embedding dimension d of a random walk (RW) taking place on a d-dimensional Sierpinski gasket fractal in its classical and dual versions. In the limit of large d the spectral dimension d_s approaches 2 such that the RW dynamics, which is governed by the d_s, is expected to behave similarly to a RW on a 2 dimensional lattice. In sharp contrast to that, we observe much richer characteristics for the RW. First, the time discrete scale invariance (DSI) phenomena cause log-periodic oscillations, which increase in amplitude for larger d. Second, the asymptotic approach to theoretically predicted power-laws of standard RW observables is significantly altered, depending on the variant of the Sierpinski gasket used (classical or dual) and on d. Furthermore, we address the suitability of standard RW observables to determine the spectral dimension d_s. This analysis is of great practical relevance and shows unexpected, puzzling results.

BP 18.4 Wed 12:00 H38

Front propagation in an $A+B \to 2A$ reaction-subdiffusion system — •Daniela Froemberg and Igor M. Sokolov — Humboldt Universität Berlin

Using the Continuous Time Random Walks approach, we derive reaction-subdiffusion equations for the irreversible autocatalytic $A + B \to 2A$ reaction, which have an integro-differential form. We show that, in contrast to the case of normal diffusion where a constant minimal velocity of the front is attained, this minimal velocity is zero in the subdiffusive case. This suggests propagation failure. Numerical simulations show that this propagation failure corresponds to a front of a stable form whose velocity decays with time. The asymptotic behavior of this velocity decay can be obtained by a crossover argument.

BP 18.5 Wed 12:15 H38

Anomalous Transport in Porous Media I: Diffusion in Carbonate Rock — •S. Afach[1], B. Biswal[2], R. Held[3], V. Khanna[1], J. Wang[1], and R. Hilfer[1,4] — [1]Institut f"ur Computerphysik, Universit"at Stuttgart, 70569 Stuttgart — [2]S.V. College, University of Delhi, New Delhi 110021 Delhi, India — [3]StatoilHydro ASA, N-7005 Trondheim, Norway — [4]Institut f"ur Physik, Universit"at Mainz, 55099 Mainz

We study diffusion through multiscale carbonate rocks using a continuum pore scale reconstruction technique. The method combines crystallite information from two dimensional high resolution images with sedimentary correlations from a three dimensional low resolution tomographic image to produce a rock sample with calibrated porosity, structural correlation and diffusion coefficient [1].

[1] B. Biswal, R. Held, V. Khanna, J. Wang, R. Hilfer, Phys.Rev.E **80** 041301 (2009)

BP 18.6 Wed 12:30 H38

Anomalous transport in porous media II: Momentum diffusion in multiscale media — •Thomas Zauner[1] and Rudolf Hilfer[1,2] — [1]Institute for Computational Physics, University of Stuttgart, 70569 Stuttgart, Germany — [2]Institute for Physics, University of Mainz, 55099 Mainz, Germany

We study viscous momentum diffusion in porous media using lattice Boltzmann simulations. A crucial transport parameter describing momentum diffusion is permeability. It is determined by the underlying stochastic geometry at the pore scale. When the geometry exhibits structure at several scales, viscous dissipation becomes scale dependent. As a consequence the permeability may become scale dependent. We use a recently introduced multiscale model for carbonate rocks [1]. Carbonate rock is known for exhibiting anomalous transport phenomena. Two methodically different lattice Boltzmann implementations, with single- and multirelaxation time collision operator, are used for numerical calculations. We find anomalous behavior in the sense of scale dependent momentum diffusion.

[1] Biswal, B. and Oren, P. E. and Held, R. J. and Bakke, S. and Hilfer, R., Phys. Rev. E, 75, 2007.

Topical Talk BP 18.7 Wed 12:45 H38

Anomalous Diffusion and Fractional Time — •R. Hilfer — Institut für Computerphysik, Universit"at Stuttgart, 70569 Stuttgart — Institut für Physik, Universit"at Mainz, 55099 Mainz

The intimate relation between anomalous diffusion in the sense of Montroll and fractional diffusion equations has been known for a long time [1]. Its fundamental importance for the theoretical understanding of anomalous diffusion processes is reflected in a growing number of applications to experimental observations [2]. Generalized fractional Riemann-Liouville derivatives of general type appear in these applications. Recently an operational calculus of Mikusiński type was developed for the resulting generalized fractional diffusion equations and their mathematical treatment [3].

[1] R. Hilfer and L. Anton, Phys.Rev.E **51**, R848 (1995)
[2] R. Hilfer, in: *Anomalous Transport: Foundations and Applications* Part I, Chapter 2, pages 17-75, Wiley-VCH, Weinheim (2007)
[3] R. Hilfer, Y. Luchko, Z. Tomovski, Fractional Calculus and Applied Analysis **12**, 299 (2009)