DY 34: Nonlinear Stochastic Systems

Time: Tuesday 14:00–15:15 Location: BH-N 333

DY 34.1 Tue 14:00 BH-N 333

Nonequilibrium dynamics of piecewise-smooth stochastic systems — •Paul Geffert and Wolfram Just — School of Mathematical Sciences, Queen Mary University of London, United Kingdom

Piecewise-smooth systems have attracted a lot of interest in the last decade; deterministic models were studied quite intensely, but the stochastic counterpart is still in its infancy.

We study the dynamics of a simple Langevin equation with dry friction subjected to coloured noise. By applying suitable approximation schemes, we obtain analytical expressions for the stationary density and the probability of sticking. Our results show very good agreement with numerical simulations. Furthermore we explore dynamical properties of the system e.g. the spectral density.

Our analysis indicates that the observed stick-slip transitions can be linked with a "critical" value of the noise correlation time.

DY 34.2 Tue 14:15 BH-N 333

A simple parameter can switch between different weak-noise-induced phenomena in a simple neuron model. — •Marius Emar Yamakou[1] and Jürgen Jost[1,2] — [1]Max Planck Institute for Mathematics in the Sciences, Inselstrasse 22, 04103 Leipzig, Germany — [2]Santa Fe Institute for the Sciences of Complexity, NM 87501, Santa Fe, USA

In recent years, several, apparently quite different, weak noise-induced resonance phenomenon have been discovered in nonlinear systems. Here, we show that at least two of them, self-induced stochastic resonance (SISR) and inverse stochastic resonance (ISR), are mathematically related by a simple parameter switch in one of the simplest models, the FitzHugh-Nagumo (FHN) neuron model. We consider a FHN model with a unique fixed point perturbed by synaptic noise. Depending on the stability of this fixed point and whether it is located to either the left or right of the fold point of the critical manifold, two distinct weak-noise-induced phenomena, either SISR or ISR, may emerge. SISR is more robust to parametric perturbations than ISR, and the coherent oscillations generated by SISR is more robust than that generated deterministically. ISR also depends on the location of initial conditions and on the time-scale separation parameter of the model equation. Our results could also explain why real biological neurons having similar physiological features and synaptic inputs may encode very different information.

DY 34.3 Tue 14:30 BH-N 333

PDE for the "first-passage-time phase" of a stochastic oscillator — •Benjamin Lindner[1], Alexander Cao[2], and Peter J. Thomas[2] — [1]Humboldt Universität zu Berlin — [2]Case Western Reserve University (Cleveland, Ohio, USA)

Phase reduction of limit cycle dynamics provides a low-dimensional representation of high-dimensional oscillator dynamics. For a deterministic dynamical system with a stable period-T limit cycle, the change of variables $\mathbf{x} \to \theta(\mathbf{x}) \in [0, 2\pi]$ such that $d\theta/dt \equiv 2\pi/T$ is well established. In contrast, for stochastic limit cycle systems, a phase reduction can be defined in several nonequivalent ways (Freund et al. *Chaos* **13**, 225 (2003), Schwabedal and Pikovsky *Phys. Rev. Lett.* **110**, 205102 (2013), Lindner and Thomas *Phys. Rev. Lett.* **113**, 254101 (2014)]. Schwabedal and Pikovsky introduced a phase for stochastic oscillators based on a foliation of the basin of attraction, with the property that the mean transit time around the cycle from each leaf to itself is uniform and developed a numerical procedure to estimate the corresponding isochrons. For robustly oscillating planar systems driven by white Gaussian noise, we establish a partial differential equation with a mixture of reflecting and jump boundary conditions that governs this phase function. We solve this equation numerically for several examples of noisy oscillators. In addition, we obtain an explicit expression for the isochron function, $\theta(\mathbf{x})$, for the rotationally symmetric case, and compare this analytical result with oscillators that have been studied numerically in the literature.

DY 34.4 Tue 14:45 BH-N 333

Quantum Hamilton equations for multidimensional problems — •Michael Beyer[1], Jeanette Köppe[1], Markus Patzold[1], Wilfried Grecksch[2], and Wolfgang Paul[1] — [1]Institute for physics, Martin-Luther-Universität Halle-Wittenberg — [2]Institute for mathematics, Martin-Luther-Universität Halle-Wittenberg

Quantum systems can be described in terms of kinematic and dynamic equations within the stochastic picture of quantum mechanics where the particles follow some conservative diffusion process. We show that the reformulation of the quantum Hamilton principle as a stochastic optimal control problem allows us to derive these quantum Hamilton equations of motion for multidimensional systems which can be seen as a generalization of Newton's equations of motion to the quantum world. In analogy to classical mechanics one encounters some similarities for quantum systems, e.g. the decoupling of the center-of-mass motion in multi-particle systems or the Kepler problem as the special case of the two-body problem where we present numerical results for the hydrogen atom.

DY 34.5 Tue 15:00 BH-N 333

Noise and Multistability in the Square Root Map — •Eoghan J. Staunton and Petri T. Piiroinen — National University of Ireland, Galway

Many real-world mechanical systems, including systems arising in engineering such as moored ships impacting a dock or rattling gears, are modelled using impact oscillators. Near low-velocity impacts the dynamics of impact oscillators can be described by a one-dimensional map known as the square root map.

In this talk we will describe the complex structure of the basins of attraction of stable periodic orbits of the square root map and how this produces sensitivity to the addition of small-amplitude white noise. In particular we focus on the effects of noise of varying amplitudes on the square root map for parameter values that lead to multistability.

We show that there is a nonmonotonic relationship between noise amplitude and the proportion of time spent in each periodic behaviour. These relationships can be explained by comparing approximations of steady-state distributions of trajectory deviations due to noise and the deterministic structures of the map. We also show that multistability can be induced by the addition of noise of an appropriate amplitude and present the mechanisms of noise-induced transitions in this case.