## MP 10: Quantum Systems, Symmetries and Scattering

Zeit: Donnerstag 14:00–15:40

MP 10.1 Do 14:00 Z6 - SR 1.012

**Derivation of Quantum Hamilton Equations of Motion** — •JEANETTE KÖPPE<sup>1</sup>, MARKUS PATZOLD<sup>1</sup>, MICHAEL BEYER<sup>1</sup>, WIL-FRIED GRECKSCH<sup>2</sup>, and WOLFGANG PAUL<sup>1</sup> — <sup>1</sup>Institut für Physik, MLU Halle-Wittenberg, Germany — <sup>2</sup>Institut für Mathematik, MLU Halle-Wittenberg, Germany

Non-relativistic quantum systems are analyzed theoretically or by numerical approaches using the Schrödinger equation. Compared to the options available to treat classical mechanical systems this is limited, both in methods and in scope. However, based on Nelson's stochastic mechanics, the mathematical structure of quantum mechanics has in some aspects been developed into a form analogous to classical analytical mechanics.

We show that finding the Nash equilibrium for a stochastic optimal control problem, which is the quantum equivalent to Hamilton's principle of least action, allows to derive two aspects: i) the Schrödinger equation as the Hamilton-Jacobi-Bellman equation of this optimal control problem and ii) a set of quantum dynamical equations which are the generalization of Hamilton's equations of motion to the quantum world. We derive their general form for the *n*-dimensional, nonstationary and the stationary case. The resulting quantum Hamilton equations of motion can be solved (numerically) without knowledge on the wave function and are analyzed for many different systems, e.g. one-dimensional double-well potential or hydrogen atom.

MP 10.2 Do 14:20 Z6 - SR 1.012 **Analytic approximation for eigenvalues of a class of**  $\mathcal{PT}$  **symmetric Hamiltonians** — •OLEG D SKOROMNIK<sup>1</sup> and ILYA D FERANCHUK<sup>2,3,4</sup> — <sup>1</sup>Max Planck Institute for Nuclear Physics, Saupfercheckweg 1, D-69117 Heidelberg, Germany — <sup>2</sup>Atomic Molecular and Optical Physics Research Group, Ton Duc Thang University, 19 Nguyen Huu Tho Street, Tan Phong Ward, District 7, Ho Chi Minh City, Vietnam — <sup>3</sup>Faculty of Applied Sciences, Ton Duc Thang University, 19 Nguyen Huu Tho Street, Tan Phong Ward, District 7, Ho Chi Minh City, Vietnam — <sup>4</sup>Belarusian State University, 4 Nezavisimosty Avenue, 220030, Minsk, Belarus

An analytical approximation for the eigenvalues of  $\mathcal{PT}$ -symmetric Hamiltonian  $\mathbf{H} = -d^2/dx^2 - (ix)^{\epsilon+2}$ ,  $\epsilon > -1$  is developed via simple basis sets of harmonic-oscillator wave functions with variable frequencies and equilibrium positions. We demonstrate that our approximation provides high accuracy for any given energy level for all values of the parameter  $\epsilon > -1$ . [1] Phys. Rev. A 96, 052102 (2017)

## MP 10.3 Do 14:40 Z6 - SR 1.012

Quantum Systems Theory Viewed from Lie Semigroups — •THOMAS SCHULTE-HERBRÜGGEN<sup>1</sup>, ROBERT ZEIER<sup>1</sup>, and GUNTHER DIRR<sup>2</sup> — <sup>1</sup>TU-München, Lichtenbergstraße 4, 85747 Garching — <sup>2</sup>Uni-Würzburg, Math. Inst., Emil-Fischer-Straße 40, 97074 Würzburg The solutions to the celebrated Kossakowski-Lindblad equation extended by coherent controls yield Markovian quantum maps. More precisely, the set of all its solutions forms a semigroup of CPTP maps taking the specific form of a *Lie semigroup* [1]. Non-trivial symmetries of these semigroups are shown to preclude accessibility in Markovian dissipative systems. This is the open-system analogue to closed systems, where triviality of (quadratic) symmetries of the Hamiltonian part suffices to decide that the system is fully controllable [2-3].

These findings are placed into a unifying Lie frame of quantum systems and control theory alongside with illustrating examples [4].

- [1] Rep. Math. Phys. 64 (2009), 93.
- [2] J. Math. Phys. 52 (2011), 113510 and 56 (2015), 081702.
- [3] Phys. Rev. A 92 (2015), 042309.

[4] Open Sys. Inf. Dynamics 24 (2017), 1750001 [in press].

MP 10.4 Do 15:00 Z6 - SR 1.012 Unitary Dilations of Discrete Quantum-Dynamical Semigroups — •FREDERIK VOM ENDE<sup>1</sup> and GUNTHER DIRR<sup>2</sup> — <sup>1</sup>TU München, Lichtenbergstr. 4 — <sup>2</sup>Universität Würzburg, Emil-Fischer-Str. 40

Quantum channels acting on trace-class operators have a well-known Stinespring dilation originally described in the Schrödinger picture.

Here, we extend this concept to the whole dynamical semigroup induced by a quantum channel yielding a unitary dilation of the semigroup. Via duality of trace-class and bounded operators one immediatly gets a similar result for quantum channels in the Heisenberg picture. Following a similar line of thought, we mathematically structure (a) the solution of discrete quantum dynamical systems and (b) several types of discrete quantum dynamical control systems.

Recently, a new experiment, MUonE, has been proposed at CERN to measure the differential cross section of the elastic scattering of high-energy muons on atomic electrons as a function of the spacelike (negative) squared momentum transfer. This measurement will provide the running of the effective electromagnetic coupling in the spacelike region and, as a result, a new and independent determination of the leading hadronic contribution to the muon g-2.

In order for this new determination to be competitive with the present dispersive one, which is obtained via timelike data, the  $\mu e$  differential cross section must be measured with statistical and systematic uncertainties of the order of 10ppm. This high experimental precision demands an analogous accuracy in the theoretical prediction.

In this work we take a step towards the calculation of the full NNLO corrections to  $\mu e$  scattering. In particular, we consider the hadronic contribution arising from virtual hadrons in the loops. Generally speaking, these corrections can be calculated via dispersion relation employing the R-ratio and timelike data for the photon vacuum polarization. However, the experiment MuonE will measure the vacuum polarization in the spacelike region. In this talk I will show how it is possible to evaluate such hadronic corrections employing only spacelike data by means of the Gegenbauer polynomials, without making use of the dispersion relation and timelike data.

Raum: Z6 - SR 1.012