## FM 21: Quantum Computation: Algorithms

Time: Monday 16:30-18:30

In this article we provide a method for fully quantum generative training of quantum Boltzmann machines with both visible and hidden units while using quantum relative entropy as an objective. This is significant because prior methods were not able to do so due to mathematical challenges posed by the gradient evaluation. We present two novel methods for solving this problem. The first proposal addresses it, for a class of restricted quantum Boltzmann machines with mutually commuting Hamiltonians on the hidden units, by using a variational upper bound on the quantum relative entropy. The second one uses high-order divided difference methods and linear-combinations of unitaries to approximate the exact gradient of the relative entropy for a generic quantum Boltzmann machine. Both methods are efficient under the assumption that Gibbs state preparation is efficient and that the Hamiltonian are given by a sparse row-computable matrix.

Quantum algorithms have the potential to outperform their classical counterparts in a variety of tasks. The realization of the advantage often requires the ability to load classical data efficiently into quantum states. However, the best known methods for loading generic data into an *n*-qubit state require  $\mathcal{O}(2^n)$  gates. This scaling can easily predominate the complexity of a quantum algorithm and, thereby, impair potential quantum advantage.

Our work demonstrates that quantum Generative Adversarial Networks (qGANs) facilitate efficient loading of generic probability distributions into quantum states. More specifically, the qGAN scheme employs the interplay of a quantum channel, a variational quantum circuit, and a classical neural network to learn the probability distribution underlying given data samples and load it into the quantum channel. The resulting quantum channel loads the learned distribution with  $\mathcal{O}(poly(n))$  gates and can, thus, enable the exploitation of quantum advantage induced by quantum algorithms, such as Quantum Amplitude Estimation.

We implement the qGAN distribution learning and loading method with Qiskit and test it using a quantum simulation as well as actual quantum processors provided by the IBM Q Experience. Furthermore, we demonstrate the use of qGANs in a quantum finance application.

 $\label{eq:FM-21.3} \begin{array}{ccc} & \text{Mon 17:15} & 2006 \\ \textbf{Quantum Algorithm for Solving Tri-Diagonal Linear Systems of Equations — •Almudena Carrera Vazquez^{1,2}, Albert Frisch<sup>3</sup>, Dominik Steenken<sup>3</sup>, Harry S. Barowski<sup>3</sup>, Ralf Hiptmair<sup>2</sup>, and Stefan Woerner<sup>1</sup> — <sup>1</sup>IBM Research, Zurich, Switzerland — <sup>2</sup>ETH Zurich, Zurich, Switzerland — <sup>3</sup>IBM Systems, Boeblingen, Germany$ 

Numerical simulations, optimisation problems, statistical analysis and computer graphics are only a few examples from the wide range of real-life applications which rely on solving large systems of linear equations. The best classical methods can approximate the solution of sparse systems in time  $poly(N, s, \kappa, \log(1/\epsilon))$ , where N denotes the number of unknowns, s the sparsity,  $\kappa$  its condition number and  $\epsilon$ the accuracy of the approximation. In 2009, A. Harrow, A. Hassidim and S. Lloyd (HHL) proposed a quantum algorithm with a running time of  $poly(\log N, s, \kappa, 1/\epsilon)$  under the assumptions of the availability of efficient methods for loading the data, Hamiltonian simulation and extracting the solution. This talk presents efficient implementations for the missing oracles and analyzes the overall performance of the algorithm. The main result presented is a novel procedure for reducing the dependency of the complexity on the error from  $1/\epsilon$  to  $log^3(1/\epsilon)$ . This method could also be used more generally to obtain a similar reduction in the gate complexity of a circuit for Hamiltonian simulation. A complete implementation of the HHL algorithm running in  $polylog(N, \kappa, 1/\epsilon)$  is given for the case of a special class of tri-diagonal

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 $\operatorname{symmetric}$  matrices.

FM 21.4 Mon 17:30 2006

Quantum Heuristic Algorithms for Hard Planning Problems from Aerospace Research — •TOBIAS STOLLENWERK<sup>1</sup>, ELISABETH LOBE<sup>2</sup>, and MÜLLER THORGE<sup>1</sup> — <sup>1</sup>German Aerospace Center (DLR), Linder Höhe, 51147 Cologne, Germany — <sup>2</sup>German Aerospace Center (DLR), Lilienthalplatz 7, 38108 Braunschweig, Germany

Quantum heuristic algorithms do not have a proven advantage over classical algorithms. However, there are indications that these approaches might outperform classical approaches for certain applications. Moreover, they are believed to work without quantum error correction and are therefore amenable to early quantum computing devices. Hard combinatorial optimization problems as they occur in logistics or traffic management are highly relevant for society and business. Even minor improvements in the solution quality can have a enormous impact in terms of costs.

We present our work on mapping and solving hard real world planning problems from aerospace research with quantum heuristic algorithms like the Quantum Approximate Optimization Algorithm (QAOA) and quantum annealing (QA). In particular, we discuss the choice of representative but small problem instances as well as the mapping of the original problem to a form compatible with the device and algorithm at hand. The latter includes various obstacles like the handling of constraints, the choice of algorithm parameters and compiling.

FM 21.5 Mon 17:45 2006

Not all entangling gates are universal for quantum computing — •JONAS HAFERKAMP, DOMINIK HANGLEITER, JENS EISERT, and JUANI BERMEJO-VEGA — Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany

It has become folklore knowledge that entanglement together with 1qubit unitaries enables full universal quantum computing. A statement along these lines was indeed proven by J. L. and R. Brylinsky: Any 2local entangling gate plus 1-qubit unitaries is universal. Here, we show that this fails to be true for 3-local entangling gates. We present an infinite family of 3-local entangling gates. Using the Schur-Weyl duality, we find a short and simple proof for the fact that access to these gates together with all 1-qubit unitaries is not universal for quantum computing.

FM 21.6 Mon 18:00 2006 Improved variational quantum algorithms for optimization problems in a quantum computer —  $\bullet$ PANAGIOTIS BARKOUTSOS<sup>1</sup>, GIACOMO NANNICINI<sup>2</sup>, ANTON ROBERT<sup>1</sup>, IVANO TAVERNELLI<sup>1</sup>, and STEFAN WOERNER<sup>1</sup> — <sup>1</sup>IBM Research - Zurich Research Lab — <sup>2</sup>IBM T.J. Watson Research Center

Recent advances in Noisy Intermediate-Scale Quantum (NISQ) computers allow us to find solutions for combinatorial optimization problems encoded in Hamiltonians via hybrid quantum/classical variational algorithms. Current approaches minimize the expectation of the problem Hamiltonian for a parameterized trial state generated in the quantum circuit. The expectation is obtained by sampling the full outcome of an ensemble of measurements of the corresponding matrix element, while the trial wavefunction parameters are optimized classically. This procedure is fully justified for quantum mechanical observables (i.e. molecular energy). However, in the case of the simulation of classical optimization problems, which yield diagonal Hamiltonians, we argue that it is more natural to aggregate the samples using a different aggregation function than the expected value. This is because our goal is simply to determine with good probability which basis state is the optimum. In this talk, we present results of the aforementioned scheme for a plethora of interesting optimization problems where we demonstrate faster convergence towards more accurate solutions.

FM 21.7 Mon 18:15 2006 Variational Quantum Eigensolver on small scale Fermi-Hubbard Models —  $\bullet$ Andreas Woitzik<sup>1</sup>, Clara Fuchs<sup>1</sup>, An-DREAS KETTERER<sup>1</sup>, PANAGIOTIS BARKOUTSOS<sup>2</sup>, IVANO TAVERNELLI<sup>2</sup>, and Andreas Buchleitner<sup>1</sup> — <sup>1</sup>Albert-Ludwigs-Universität Freiburg, Freiburg im Breisgau, Deutschland — <sup>2</sup>IBM Research -

## ${\it Zurich, R\" uschlikon, Schweiz}$

Algorithms processing quantum information seem to be a good candidate for optimisation processes as well as simulations of quantum systems. Due to limited capabilities of currently available quantum hardware the application of these algorithms is restricted to proof-ofprinciple examples. We elaborate on a quantum-classical variational algorithm applied to small scale Fermi-Hubbard models. The impact of noise on the algorithm is discussed and we present its numerically achieved accuracy depending on the complexity of the quantum circuit by applying an argument based on the Solovay-Kitaev theorem. We find that the algorithm's noise resilience strongly depends on the classical optimisation scheme which is used.