

## MP 12: Quantum Mechanics

Time: Wednesday 17:50–18:15

Location: H7

MP 12.1 Wed 17:50 H7

**Stability of quantum inequalities under scattering** —  
•HENNING BOSTELMANN<sup>1</sup>, DANIELA CADAMURO<sup>2</sup>, and GANDALF  
LECHNER<sup>3</sup> — <sup>1</sup>University of York, Department of Mathematics, York  
YO10 5DD, United Kingdom — <sup>2</sup>Universität Leipzig, Institut für The-  
oretische Physik, Brüderstraße 16, 04103 Leipzig — <sup>3</sup>School of Math-  
ematics, Cardiff University, Senghennydd Road, CF24 4AG Cardiff,  
United Kingdom

Certain physical quantities that yield positive values in classical me-  
chanics can have negative expectation values in quantum theory (e.g.,  
the probability flux in the quantum backflow effect, or the averaged  
energy density in field theories). However, they typically possess a

lowest negative eigenvalue. In other words, positive observables in  
classical theory often “quantize” to operators that are not necessarily  
positive, but bounded below (“quantum inequalities”). Here we in-  
vestigate whether, for one quantum mechanical particle, such bounds  
are stable when the dynamics is perturbed by a scattering potential.  
This boils down to the question how fast the Møller operator  $\Omega$  ap-  
proaches the identity at high energies; more quantitatively, whether  
 $\|(\Omega - 1)(1 + H_0)^\beta\| < \infty$  for suitable  $\beta > 0$ , where  $H_0$  is the free Hamil-  
tonian. We derive such bounds under generic assumptions on the free  
Hamiltonian and the scattering potential. In particular,  $0 < \beta \leq 1/2$   
is allowable for the Schrödinger Hamiltonian – independent of space  
dimension, and even in the matrix-valued case, i.e., when adding inner  
degrees of freedom.