

DY 4: Pattern Formation, Delay and Nonlinear Stochastic Systems

Time: Monday 9:30–12:30

Location: ZEU 250

DY 4.1 Mon 9:30 ZEU 250

Spiral waves within a bistability parameter region of an excitable medium — ●VLADIMIR ZYKOV and EBERHARD BODENSCHATZ — Max Plank Institute for Dynamics and Self-Organization, D-37077, Goettingen, Germany

Spiral waves are a well-known and intensively studied dynamic phenomenon in excitable media of various types. Most studies have considered an excitable medium with a single stable resting state. However, spiral waves can be maintained in an excitable medium with bistability. Our calculations, performed using the widely used Barkley model, clearly show that spiral waves in the bistability region exhibit unique properties. For example, a spiral wave can either rotate around a core that is in an unexcited state, or the tip of the spiral wave describes a circular trajectory located inside an excited region. The boundaries of the parameter regions with positive and negative cores have been defined numerically and analytically evaluated. It is also shown that the creation of a positive or *negative* core may depend on the initial conditions, which leads to hysteresis of spiral waves.

DY 4.2 Mon 9:45 ZEU 250

Band Pattern Formation in a Suspension of Red Blood Cells During Centrifugation in a Percoll Density Gradient — ●FELIX MAURER, THOMAS JOHN, CHRISTIAN WAGNER, and ALEXIS DARRAS — Dynamics of Fluids, Experimental Physics, Saarland University, 66123 Saarbrücken, Germany

Percoll is a suspension of silica nanoparticles often used to establish density gradients and separate biological matter in centrifugation protocols. When red blood cells (RBCs) sediment in a Percoll medium, they form patterns of discrete bands. While this is a popular approach for RBC age separation, the mechanisms involved in band formation were unknown. In a series of experiments we could show that the formation of those patterns could be explained by cell aggregation. We developed a new continuum model to describe the volumetric RBC density under the influence of attractive pair interaction. Our numerical solutions are characterized by pattern formation and transitions between the equilibrium states depending on aggregation energy and initial volumetric RBC concentration.

DY 4.3 Mon 10:00 ZEU 250

A missing amplitude equation — ●TOBIAS FROHOFF-HÜLSMANN¹ and UWE THIELE^{1,2} — ¹Institute of Theoretical Physics, WWU Münster — ²Center for Nonlinear Science (CeNoS), WWU Münster

Amplitude (or envelope) equations describe the spatiotemporal dynamics of the essential linear mode(s) in the vicinity of a stability threshold and represent universal equations for spatially extended systems [3]. They are determined by the type of linear instability, the symmetries and whether or not conservation laws are present [5, 6]. For systems without conservation laws these equations are well studied, e.g. the complex Ginzburg-Landau equation [1]. However, the presence of conservation laws is highly relevant for a wide spectrum of pattern forming systems, e.g. for certain reaction diffusion (RD) systems [2, 4]. Here, we review the basic types of linear instabilities in the presence of conservation laws and show that there are relevant cases for which the amplitude equation is still unknown. We focus on such a missing case and derive an amplitude equation relevant for practically important RD systems.

- [1] I. S. Aranson and L. Kramer. *Rev. Mod. Phys.*, 74:99-143, 2002.
- [2] C. Beta, N. S. Gov, and A. Yochelis. *Cells*, 9:1533, 2020.
- [3] M. C. Cross and P. C. Hohenberg. *Rev. Mod. Phys.*, 65:851-1112, 1993.
- [4] J. Halatek and E. Frey. *Nature Phys.*, 14:507-514, 2018.
- [5] P. C. Matthews and S. M. Cox. *Nonlinearity*, 13:1293-1320, 2000.
- [6] F. Bergmann, L. Rapp, and W. Zimmermann. *Phys. Rev. E*, 98:020603, 2018.

DY 4.4 Mon 10:15 ZEU 250

The universal CHEOPS, the path to it, and applications — ANDRE FÖRTSCH and ●WALTER ZIMMERMANN — Theoretische Physik, Universität Bayreuth

Solutions to fundamental questions in the field of nonequilibrium phase transitions are presented. What are the 'generic transport equations for oscillatory phase separation' (GTOPS) in systems described by con-

served fields? GTOPS cover both classical and oscillatory phase separation. But what is the universal equation for oscillatory phase separation, i.e., the counterpart of the famous universal complex Ginzburg-Landau equation (cGLE) for an unconserved order parameter [1]? It is the 'Cahn-Hilliard model extended to oscillatory phase separation' (CHEOPS) that includes the model in [2] as a special case. By generalizing methods from [3-6] CHEOPS is derived from GTOPS or even from a chemotaxis model for two species. Examples of surprising solutions of GTOPS and CHEOPS (patterns) are presented and some of them are also illustrated by a so-called minimal model (MIMO).

- [1] I. Aranson, L. Kramer, *Rev. Mod. Phys.* 74, 99 (2002)
- [2] W. Zimmermann, *Physica A* 237, 575 (1997)
- [3] F. Bergmann et al., *Phys. Rev. E* 98, 020603(R) (2018)
- [4] L. Rapp et al., *Eur. Phys. J E* 42, 57 (2019)
- [5] F. Bergmann, W. Zimmermann, *PLoS ONE* 14, e0218328 (2019)
- [6] F. J. Thomsen, L. Rapp, F. Bergmann, W. Zimmermann, *New J. Phys.* (FT) 23, 042002 (2021)

DY 4.5 Mon 10:30 ZEU 250

Quasi-steady interface flows in simple reaction-diffusion systems — ●TOBIAS ALEXANDER ROTH, HENRIK WEYER, and ERWIN FREY — Arnold Sommerfeld Center for Theoretical Physics and Center for NanoScience, Department of Physics, Ludwig-Maximilians-Universität München, München, Germany

Intracellular protein patterns are essential features of living systems. A well-studied framework for describing simple protein systems are 2-component reaction-diffusion systems that preserve mass (2cMCRD). These genuine non-equilibrium systems can not be written in terms of a variational approach: there is neither a free energy nor a classical surface tension. Interestingly, it was found that the long-term evolution of these reaction-diffusion systems, however, is phenomenologically similar to the interface dynamics of phase-separating thermodynamic systems.

Here we show that an interface line in 2cMCRD systems obeys a flow, that interpolates between two paradigmatic limits: the two-sided Mullins-Sekerka flow and the area-preserving geodesic curvature flow. This generalised flow conserves area and minimises the interface length. One can tune its character by the time scale of diffusive mass redistribution compared to reactive turnover.

15 min. break

DY 4.6 Mon 11:00 ZEU 250

Amplitude expansion of the phase-field crystal model on deformable surfaces — ●LUCAS BENOIT-MARÉCHAL, MARCO SALVALAGLIO, INGO NITSCHKE, and AXEL VOIGT — Institute of Scientific Computing, TU Dresden, Dresden, Germany

The Phase Field Crystal (PFC) model describes lattices at diffusive timescales but atomic lengthscales, thus requiring subatomic resolution meshes. To remedy this restriction, the complex amplitude expansion (APFC) was developed, whereby the amplitude of the density fluctuations is modeled instead of the density itself, enabling simulations at mesoscales that retain atomistic features.

We extend the two-dimensional APFC model to include out-of-plane displacements in order to study the coupling between crystal defects and surface deformation, paving the way for applications such as the topological tuning of mechanical properties of crystalline sheets.

To validate our model, we compare representative settings with atomistic simulations from the PFC model and Molecular Dynamics and find, within certain limits that we discuss, excellent agreement between all models.

DY 4.7 Mon 11:15 ZEU 250

Laminar chaos in systems with quasiperiodic delay — ●DAVID MÜLLER-BENDER¹ and GÜNTER RADONS^{1,2} — ¹Institute of Physics, Chemnitz University of Technology, 09107 Chemnitz, Germany — ²ICM - Institute for Mechanical and Industrial Engineering, 09117 Chemnitz, Germany

A new type of chaos called laminar chaos was found in singularly perturbed dynamical systems with periodic time-varying delay [Phys. Rev. Lett. 120, 084102 (2018)]. It is characterized by nearly constant laminar phases, which are periodically interrupted by irregular bursts,

where the intensity levels of the laminar phases vary chaotically from phase to phase. In this paper, we demonstrate that laminar chaos can also be observed in systems with quasiperiodic delay, where we generalize the concept of conservative and dissipative delays to such systems. It turns out that the durations of the laminar phases vary quasiperiodically and follow the dynamics of a torus map in contrast to the periodic variation observed for periodic delay. Theoretical and numerical results indicate that introducing a quasiperiodic delay modulation into a time-delay system can lead to a giant reduction of the dimension of the chaotic attractors. By varying the mean delay and keeping other parameters fixed, we found that the Kaplan-Yorke dimension is modulated quasiperiodically over several orders of magnitudes, where the dynamics switches quasiperiodically between different types of high- and low-dimensional types of chaos.

Details can be found in the preprint [arXiv:2210.04706 (2022)].

DY 4.8 Mon 11:30 ZEU 250

Pulse generation in opto-electronic neurons with time-delayed feedback — ●JONAS MAYER MARTINS¹, SVETLANA V. GUREVICH¹, and JULIEN JAVALOYES² — ¹Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Str. 9 and Center for Nonlinear Science (CeNoS), University of Münster, Corrensstrasse 2, 48149 Münster, Germany — ²Departament de Física and IAC-3, Universitat de les Illes Balears, C/ Valldemossa km 7.5, 07122 Mallorca, Spain

We study a neuromorphic circuit composed of a nano resonant tunneling diode (RTD) operated in the excitable regime, where the diode generates, when triggered, an all-or-nothing electrical response pulse. This pulse is fed into a nano laser diode (LD), which in turn emits an optical pulse that is re-injected with time delay back into the RTD. Our theoretical analysis of this time-delayed opto-electronic nonlinear system describes how such neuron-like excitability can lead to sustained periodic pulsations due to the time-delayed feedback. We derive a bifurcation diagram through numerical continuation, unveiling the rich dynamics of the system. Furthermore, direct numerical simulations of the RTD-LD reveal emerging solitons that may serve as memory for information. Opto-electronic neurons like the RTD-LD are particularly interesting because they allow for fast computations at very low energy consumption and are therefore promising candidates for new computational architectures that mimic the brain.

DY 4.9 Mon 11:45 ZEU 250

Spontaneous vortex formation by microswimmers with retarded attractions — XIANGZUN WANG¹, ●PIN-CHUAN CHEN², KLAUS KROY², VIKTOR HOLUBEC³, and FRANK CICHOS¹ — ¹Peter Debye Institute for Soft Matter Physics, Leipzig University, 04103 Leipzig, Germany — ²Institute for Theoretical Physics, Leipzig University, Postfach 100 920, 04009 Leipzig, Germany — ³Department of Macromolecular Physics, Faculty of Mathematics and Physics, Charles University, 18000 Prague, Czech Republic

In recent experiments done in the Molecular Nanophotonics Group in the Peter Debye Institute, thermophoretic microswimmers are observed to self-assemble into bi-stable orbital modes due to retarded attractive interactions.

A single agent which is attracted to an immobilized target with a time delay can be described by a time-local overdamped Langevin equation with a potential determined by time delay, and the transition

in between the two stable modes is well predicted by Kramers' escape rate. Simulations of multiple agents attracted to one target also show that the collective behavior can be reduced to a one-agent description; however, the experiments (with up to 16 agents) show otherwise. The discrepancy between the results are attributed to additional effects in experiments.

We further show results of extended simulations with larger number of agents, which demonstrate two transitions depending solely on the time delay.

DY 4.10 Mon 12:00 ZEU 250

Stochastic pH oscillator confined to lipid vesicles — ●ARTHUR STRAUBE¹, STEFANIE WINKELMANN¹, and FELIX HÖFLING^{2,1} — ¹Zuse Institute Berlin — ²Institut für Mathematik, Freie Universität Berlin

We study an urea-urease-based pH oscillator confined to lipid vesicles serving as an open reactor [1,2]. In contrast to conventional pH oscillators in closed reactors, the exchange with the vesicle exterior periodically resets the pH clock that switches the system from acid to basic, resulting in self-sustained oscillations. Stochastic simulations for microscopically small vesicles predict a significant statistical variation of the oscillation period. Although the mean period remains remarkably robust for vesicle sizes down to nearly 200 nm, the periodicity of the rhythm is gradually destroyed for smaller vesicles [1]. We analyze the structure of the limit cycle, which controls the dynamics for giant vesicles and dominates the strongly stochastic oscillations in small vesicles of submicrometer size. We derive reduced two-variable models, amenable to analytic treatments, and show that the accuracy of predictions, including the period of oscillations, is highly sensitive to the choice of the reduction scheme [2]. The accurate description of a single pH oscillator is crucial for rationalizing experiments and understanding communication of vesicles and synchronization of rhythms.

[1] A. Straube, S. Winkelmann, C. Schütte, F. Höfling, J. Phys. Chem. Lett. **12**, 9888 (2021). [2] A. Straube, S. Winkelmann, F. Höfling, ZIB Report 22-21 (2022), preprint (DOI: 10.12752/8817).

DY 4.11 Mon 12:15 ZEU 250

Sampling from the rule 150 fractal through an iterated stochastic process — ●JENS CHRISTIAN CLAUSSEN — University of Birmingham, UK

A widely known, but surprising way of sampling points from the Sierpinski fractal is through an iterated stochastic process where in each time step one of three operators is applied, which can be interpreted from their number representation, or as a geometric operation. While the Sierpinski fractal can also be generated by the rule 90 elementary cellular automaton (ECA), the ECA rule 150 generates a fractal pattern with a 2-step self-similarity resembling a generalization of a Fibonacci iteration [1]. Here we show that the rule 150 fractal can be generated without a 2-step iteration. We introduce a set of 6 operators, which allow to generate the rule 150 fractal from a stochastic process. We show that these 6 operators can be reduced to 4 operators, by adding one operator to the 3 operators from the rule 90 case. The operators for the rule 150 can be interpreted both from their number representation and geometrically. Further each point of the rule 150 fractal can be represented by a any base-6 number, or by a 4-letter symbolic sequence with a grammar restriction.

[1] Jens Christian Clausen, J. Math. Phys. **49**, 062701 (2008)