# DY 45: Poster: Nonlinear Dynamics, Pattern Formation and Networks

Time: Thursday 13:00–16:00 Location: P1

DY 45.1 Thu 13:00 P1

**A systematic approximation scheme mapping systems with time delays to sets of ordinary differential equations** — •Daniel Henrik Nevermann and Claudius Gros — Institut für Theoretische Physik, Goethe-Universität Frankfurt, Deutschland

Mathematically, delayed differential equations evolve in infinite dimensional state spaces. It is hence conceivable that time-delayed systems can be approximated by a set of $N+1$ ordinary differential equations, with the trajectory of the primary variable converging to the solution of the original time-delayed system when $N \to \infty$. We show that this program can be carried out using sequences of time-delay kernels related to discrete gamma distributions.

We present several analytical and numerical results for the proposed approximation scheme, finding that the instability of fixed points due to increasing time delays is captured accurately already for $N \sim 10$. For the Mackey-Glass system we find that the locus of a limit-cycle doubling are recovered in good approximation only for substantially larger $N \sim 10^2 - 10^3$, with the transition to chaos requiring an even larger state space. In general, we find that relative approximation errors scale as $1/N$. In addition, we discuss how the approximation proposed can be applied to the case of distributed time delays.

It is in general an approximation to model a given experimental protocol by a dynamical system characterized by a single time delay $T$. Using a distribution of time delays peaked at $T$, with width $\sim 1/N$, can hence be argued to provide a more accurate description of real-world non-Markovian processes.

DY 45.2 Thu 13:00 P1

**Preprocessing algorithms for the estimation of ordinary differential equation models with polynomial nonlinearities** — •Oliver Strebel — Angelstr. 17, 75392 Deckenpfronn

The data analysis task of determining a model for an ordinary differential equation (ODE) system from given noisy solution data is addressed. Based on a previously published parameter estimation method for ODE models [1] four related model estimation algorithms were developed. The algorithms are tested for over 20 different polynomial ordinary equation systems comprising 60 equations at various noise levels. Two algorithms frequently compute the correct model [2]. They are compared to the prominent SINDy-family for those SINDy-algorithms that have simple default hyperparameters [3]. A novel and successful method for determining the parameter of Tikhonov regularization when calculating numerical differentials is also presented.

[1] O. Strebel: http://dx.doi.org/10.1016/j.chaos.2013.08.015
[2] O. Strebel: https://osf.io/89djt/
[3] S. Brunton et al: http://dx.doi.org/10.1073/pnas.1517384113

DY 45.3 Thu 13:00 P1

**Testing Jump-Diffusion in Epileptic Brain Dynamics: Impact of Daily Rhythms** — •Jutta G. Kurth[1,2], Klaus Lehnertz[2], and Thorsten Rings[2] — [1]Georg-August-Universität Göttingen — [2]Rheinische Friedrich-Wilhelms-Universität Bonn

Stochastic approaches to complex dynamical systems have recently provided broader insights into spatial-temporal aspects of epileptic brain dynamics. Stochastic qualifiers based on higher-order Kramers-Moyal coefficients derived directly from time series data indicate improved differentiability between physiological and pathophysiological brain dynamics. It remains unclear, however, to what extent stochastic qualifiers of brain dynamics are affected by other endogenous and/or exogenous influencing factors. Addressing this issue, we investigate multi-day, multi-channel electroencephalographic recordings from a subject with epilepsy. We apply a recently proposed criterion to differentiate between Langevin-type and jump-diffusion processes and observe the type of process most qualified to describe brain dynamics to change with time. Stochastic qualifiers of brain dynamics are strongly affected by endogenous and exogenous rhythms acting on various time scales*ranging from hours to days. Such influences would need to be taken into account when constructing evolution equations for the epileptic brain or other complex dynamical systems subject to external forcings.

Keywords: diffusion process; jump-diffusion process; time series analysis; brain; epilepsy; biological rhythms

DY 45.4 Thu 13:00 P1

**Temporal localized states and square waves in semiconductor micro-resonators with strong time delayed feedback** — •Elias R. Koch[1,2], Thomas Seidel[1], Julien Javaloyes[2], and Svetlana V. Gurevich[1] — [1]Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Str. 9, 48149 Münster, Germany — [2]Departament de Física & IAC-3, Universitat de les Illes Balears, C/ Valldemossa km 7.5, 07122 Mallorca, Spain

Recent works demonstrated the promising potential of injected micro-resonators enclosed into external cavities as high-power, tunable sources of Frequency Combs in the near infra-red. It was shown that the natural modeling approach consists in using singularly perturbed time delayed systems. Departing from former studies that considered a single intensity dependent refractive index (i.e. Kerr nonlinearity) we explore in this contribution the impact of a semiconductor Quantum-Well as the nonlinear element. A first principle model for the optical response is employed which allows to explore the influence of the detuning with respect to the band-gap. We show that this extended model predicts the existence of a bistable set of bright and dark temporally localized states as well as square-waves, with a periodic of twice the delay in the case of antiresonant optical feedback.

Finally, in order to clarify the influence of the second and third order chromatic dispersion and of the frequency dependence of the quantum-well response, we perform a multiple time-scale analysis in the so-called good cavity limit. The resulting normal form PDE shows a good agreement with the original, first principle, time delayed model.

DY 45.5 Thu 13:00 P1

**Antipersistent random walks in time-delayed systems** — •Tony Albers[1], David Müller-Bender[1], and Günter Radons[1,2] — [1]Institute of Physics, Chemnitz University of Technology, Chemnitz, Germany — [2]Institute for Mechanical and Industrial Engineering, Chemnitz, Germany

In this contribution, we show that the occurrence of chaotic diffusion in a typical class of time-delayed systems with linear instantaneous and nonlinear delayed term can be well described by an antipersistent random walk. We numerically investigate the dependence of all relevant quantities characterizing the random walk on the strength of the nonlinearity and on the delay. With the help of analytical considerations [1], we show that for a decreasing nonlinearity parameter the resulting dependence of the diffusion coefficient is well described by Markov processes of increasing order.

[1] Tony Albers, David Müller-Bender, and Günter Radons, Phys. Rev. E **105**, 064212 (2022)

DY 45.6 Thu 13:00 P1

**Advection dependent pulse dynamics** — •Adrian Misselwitz[1], Susanne Lafon[2], Jean-Daniel Julien[2], and Karen Alim[1,2] — [1]School of Natural Sciences, Technische Universität München — [2]Max-Planck-Institut für Dynamik und Selbstorganisation, Göttingen

Models of pulse formation in nerve conduction have provided manifold insight not only into neuronal dynamics but also the non-linear dynamics of pulse formation in general. Recent observation of neuronal electro-chemical pulses also driving mechanical deformation of the tubular neuronal wall and thereby generating ensuing cytoplasmic flow now question the impact of flow on the electro-chemical dynamics of pulse formation. We, here, theoretically investigate the classical Fitzhugh-Nagumo model now accounting for advective coupling between the pulse propagator typically describing membrane potential and here triggering mechanical deformations and, thus, governing flow magnitude, and the pulse controller, a chemical species advected with the ensuing fluid flow. Employing analytical calculations and numerical simulations we find, that advective coupling allows for a linear control of pulse width while leaving pulse velocity unchanged. We therefore uncover an independent control of pulse width by fluid flow coupling.

DY 45.7 Thu 13:00 P1

**(Broken) gradient-dynamics description of reactive thin liquid films** — •Florian Voss, Fenna Stegemerten, and Uwe Thiele — Institut für Theoretische Physik, Westfälische Wilhelms-Universität Münster, Wilhelm-Klemm-Str. 9, 48149 Münster, Germany

After reviewing the gradient dynamics formulation of chemical reactions, applied e.g. in [1] to reacting phase-separating systems, we apply the concept to thin liquid films and shallow drops that are either covered by reactive surfactants [2] or react with the solid substrate [3]. Next we discuss how the breaking of the variational form by imposed fluxes results in intricate spatio-temporal dynamics of the film/drop and reactant density profiles. As an example we consider a simple model for oscillatory behaviour in droplets of slime mould.

[1] D. Zwicker, Current Opinion in Colloid & Interface Science, 61, 101606 (2022),

[2] A. Pereira, P. M. J. Trevelyan, U. Thiele, and S. Kalliadasis, Phys. Fluids 19, 112102 (2007),

[3] K. John, M. Bär, and U. Thiele, Eur. Phys. J. E 18, 183 (2005).

DY 45.8 Thu 13:00 P1

**Coupling short-range signaling and tissue mechanics for biological pattern formation** — •Valèria Ribelles Pérez, Stephan Kremser, Mareike Bojer, Sabina Orazov, and Ulrich Gerland — Physics Department, Technical University of Munich

Pattern formation phenomena are ubiquitous in natural and synthetic multicellular systems. Both mechanical forces and biochemical interactions between cells play key roles in tissue dynamics. While much is known about these physical and biochemical processes separately, their interplay is still poorly understood. Here, we focus on short-range signaling between cells, modelled by cellular automata, coupled to a vertex model incorporating mechanical interactions, to investigate patterning principles during tissue homeostasis and growth. We test the modelling framework in the context of salt-and-pepper-like patterns that arise for instance in epithelial tissues.

DY 45.9 Thu 13:00 P1

**Robust, precise, and modular solutions to the French flag problem in two dimensions without global signaling** — •Lukas Zett, Stephan Kremser, Gabriel Vercelli, and Ulrich Gerland — Technical University of Munich

The formation of axial patterns with broad regions in multicellular systems has been conceptualized by Wolpert in his famous French flag problem. Both of Wolpert's proposed solutions, the balancing and gradient model, utilize long-range signaling between cells. Models relying on short-range signaling, based on cellular automata (CA) rules as modeling tool, have also computationally been shown to successfully solve the French flag problem in one dimension (1D). Here, we extend these models to two spatial dimensions (2D) to investigate whether the 1D solutions can be generalized to the 2D case and to search for novel solutions existing only in 2D. We dissect the 2D problem into two coupled, 1D pattern formation processes along and perpendicular to the axis of the French flag. Using evolutionary algorithms and consensus procedures as well as engineering approaches, CA dynamics which solve the French flag problem are found. We show that these solutions form more precise patterns and are in general more robust than their 1D counterparts, while still being able to scale with system size. Depending on the desired robustness and precision of the solution, different patterning modules along the two axes can be combined. Using the regulatory logics of these underlying modules could therefore serve as a basis for the design of synthetic patterning systems with a range of different specifications.

DY 45.10 Thu 13:00 P1

**Kinetic Monte Carlo Model for Computing Functionalities in Nanoparticle Networks** — •Jonas Mensing[1] and Andreas Heuer[2] — [1]Institut für Physikalische Chemie, WWU Münster — [2]Institut für Physikalische Chemie, WWU Münster

We want to achieve reconfigurable computational unctionality in a nanoparticle network for energy efficient machine learning applications. Previous research has shown that disordered networks of functionalized gold nanoparticles can be configured to behave like Boolean logic gates and binary classifiers. In this regard, gold nanoparticles serve as switchable single-electron transistors, while organic molecules connecting the nanoparticles act as tunable tunnel barriers. The resulting network is then placed within an array of electrodes that manipulate the charge and potential landscape of the network to evolve the system into its desired emergent functionality. In total, the network is able to mimic the mechanism of a brain-like neural network. The theoretical underpinning of these networks is investigated with a highly optimized physical model and subsequent simulations. The model is able to simulate the charge transport within the network stochastically, i.e. with a kinetic Monte Carlo approach. Requirements for various computing functionalities such as Boolean logic are examined. Besides graph theory and data-driven tools allow mapping network and electrode properties to the appearance of computational functionalities. The simulations are carried out in close comparison with corresponding experiments.

DY 45.11 Thu 13:00 P1

**Randomised mixed labyrinth fractals** — •Janett Prehl[1], Ligia Loretta Cristea[2], and Daniel Dick[1] — [1]Technische Universität Chemnitz, Chemitz, Germany — [2]Technische Universität Graz, Graz, Austria

Fractals, introduced by Benoit Mandelbrot in the early 1980s, allow the analysis of physical properties of natural geometries and structures in non-integer dimensions. It has been shown recently, that utilizing fractals structures, for instance for gas sensors made of carbon nanotubes increase their efficiency or give new insights to complex quantum phenomena. Here, we are interested how the effect of randomness, as observed in real materials, alter the topology and thus dynamics of the resulting fractal structures in comparison to the pure cases. We focus on a special class of Sierpinski carpets, i.e., the labyrinth fractals [1], that can be used for dendritic networks or porous materials. Therefore, we mix to fractal patterns, with different properties, i.e., shortest path and random walk dimension, randomly together at different mixing ratios. Surprisingly we found that even in cases where the initial patterns exhibit the same non-integer dimensions the resulting randomised fractals give a different property [2].

[1] L.L. Cristea and B. Steinsky, *Proc. Edinburgh Math. Soc.* **54.4** (2011) 329.

[2] J. Prehl, D. Dick, and L.L. Cristea, to be submitted to *Fractals* (2023).

DY 45.12 Thu 13:00 P1

**How Can Cell-Like Inflated Shells Control Their Shape? —A Stability Analysis** — •Paul Nemec and Ulrich Gerland — Physics Department, Technical University of Munich

This work follows a long history of studying how biological organisms arrive at and maintain their shape [1]. Inspired by the question of how E. coli maintain their cylindrical shape during growth [2], we study the growth of pressurised cell-like structures. The model is this: a cell is an inflated elastic shell, where internal and fine grained details are neglected. Growth is the time evolution of the reference or undeformed configuration of the cell, which may depend on geometric and mechanical properties like curvature and stress. Growth must be local and invariant under translations and rotations of the entire cell. Under these constraints, how can cells robustly achieve simple target geometries like a sphere given arbitrarily perturbed initial conditions? This poster presents some initial insights.

[1] Goriely, A. The Mathematics and Mechanics of Biological Growth, Springer (2017).

[2] Amir, A., van Teeffelen, S. Getting into shape: How do rod-like bacteria control their geometry?. *Syst Synth Biol* **8**, 227–235 (2014).

DY 45.13 Thu 13:00 P1

**Stimulating self-optimisation of flow networks for transport** — •Swarnavo Basu and Karen Alim — School of Natural Sciences, Technical University of Munich, Germany

Flow transport in networks is ubiquitous in biology (e.g. blood vasculature) and engineering (e.g. porous media). Many biological networks are adaptive and can self-organise in response to external stimuli. They homogenise flow to achieve optimal perfusion and a uniform flow of chemicals across the network. In contrast, engineered networks of random media have heterogeneous flow velocity distributions across the network. Self-organising engineered networks that can homogenise flow will have many applications, ranging from microfluidic networks for cooling batteries and chemical reactors to *in vitro* vasculature for perfusing tissues and implants. We propose a model of a network whose tube radii can be controlled using periodic inflows of pulses of an eroding agent that erodes the network's walls. We observe that such networks self-organise in response to the eroding agent, leading to a homogenised flow. This provides a framework for engineering networks that can self-organise to achieve optimal perfusion.

DY 45.14 Thu 13:00 P1

**Fixation probabilities in network structured meta-**

**populations** — •Sedigheh Yagoobi[1] and Arne Traulsen[2] — [1]Max-Planck institute for evolutionary biology — [2]Max-Planck Institute for Evolutionary Biology

The effect of population structure on evolutionary dynamics is a long-lasting research topic in evolutionary ecology and population genetics. Evolutionary graph theory is a popular approach to this problem, where individuals are located on the nodes of a network and can replace each other via the links. We study the effect of complex network structure on the fixation probability, but instead of networks of individuals, we model a network of sub-populations with a probability of migration between them. We ask how the structure of such a meta-population and the rate of migration affect the fixation probability. Many of the known results for networks of individuals carry over to meta-populations, in particular for regular networks or low symmetric migration probabilities. However, when patch sizes differ we find interesting deviations between structured meta-populations and networks of individuals. For example, a two patch structure with unequal population size suppresses selection for low migration probabilities.

DY 45.15 Thu 13:00 P1

**Ising model with variable spin/agent strengths on graphs** — Mariana Krasnytska[1,2,3], Yurij Holovatch[1,2,4], •Bertrand Berche[2,3], and Ralph Kenna[2,4] — [1]ICMP, NAS of Ukraine, Lviv, Ukraine — [2]L4 Collaboration Leipzig-Lorraine-Lviv-Coventry — [3]Université de Lorraine, Nancy, France — [4]Coventry University, UK

We consider a generalization of the Ising model in which individual spin strengths can vary [1]. The model describes the ordering in systems comprised of agents which, although matching in their binarity (i.e., maintaining the iconic Ising features of spin 'up'/'down', 'yes'/'no'), differ in their strengths. With inhomogeneous physical systems in mind, but also anticipating interdisciplinary applications, we present the model on graph structures of varying degrees of complexity: complete graph, Erdös-Rényi graph, and on a scale-free network. This allows us to explore the interplay of two types of randomness: individual strengths of spins or agents and collective connectivity between them. We find the delicate interplay between variable properties of nodes and interactions between them leads to new universality classes.

[1] M. Krasnytska, B. Berche, Yu. Holovatch, R. Kenna, J. Phys. Complex., 1 (2020) 035008; Entropy, 23(9) (2021) 1175.

DY 45.16 Thu 13:00 P1

**Homeostatic plasticity in a minimal model for brain criticality** — •Marco Schmidt and Stefan Bornholdt — Institut für Theoretische Physik, Universität Bremen

The 'criticality hypothesis' asserts that real-world neural networks operate near a critical phase transition. Experimental evidence exists and numerous models studying the possible underlying mechanisms accumulated during the last 20 years.

Early models based on simple threshold networks tune to a critical connectivity $K = 2$, which is not a realistic value when compared to real-world neural networks.

However, a phase transition in high degree threshold networks using the inhibition to excitation ratio as a control parameter does exist [1], as well as a corresponding self-organized critical toy model [2]. It features an adaptive threshold network, self-tuning to the critical inhibition to excitation ratio by using an activity based rewiring process that results in a highly clustered network and reaches criticality independent of $K$.

Here we present a new version of the model, incorporating a simple homoestatic plasticity mechanism as it appears in biological systems.

[1] L. Baumgarten, S. Bornholdt, Critical excitation-inhibition balance in dense neural networks, Phys. Rev. E 100, 010301 (2019).
[2] L. Baumgarten, S. Bornholdt, A toy model for brain criticality: self-organized excitation/inhibition ratio and the role of network clustering, arXiv:2202.03330.