

DY 4: Focus Session: Nonlinear Dynamics and Stochastic Processes – Advances in Theory and Applications I

Deterministic chaos and stochastic processes are often seen as opposites. However, not only do they share the aspect of being mechanisms for irregular temporal fluctuations, but more importantly, the theory of stochastic processes has helped to understand and quantify many aspects of chaos. Deterministic diffusion, intermittency, long-range temporal correlations can be generated by simple deterministic systems, but are conveniently characterized by concepts of stochastic processes. In both classes of systems, the inclusion of time delays in feedbacks through memory kernels has introduced additional phenomena, and more recently non-normalizable distributions have been found as causes of ageing. While a unified approach to chaos and stochastics is fascinating and satisfying from a theoretical point of view, it also has surprisingly strong application relevance. Examples include the study of turbulence, the nonlinear dynamics of wind turbines, industrial processes for metal milling and turning, laser dynamics, cardiac dynamics, and neuronal systems.

Organized by Robert Magerle (Chemnitz) and Holger Kantz (Dresden)

Time: Monday 9:30–13:00

Location: H43

DY 4.1 Mon 9:30 H43

Welcome and Remarks — ●ROBERT MAGERLE¹, HOLGER KANTZ², and THEO GEISEL³ — ¹Technische Universität Chemnitz, Institut für Physik, 09126 Chemnitz, Germany — ²Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany — ³Max Planck Institute for Dynamics and Self-Organization, 37077 Göttingen, Germany

Welcome and introductory remarks.

DY 4.2 Mon 9:45 H43

Invited Talk **Physical application of infinite ergodic theory** — ●ELI BARKAI — Phys. Dept. Bar-Ilan University, Ramat-Gan, Israel

Norm conserving dynamical mappings, for example the Pomeau Manneville scenario for intermittency, exhibit either a normalized invariant density or an infinite (non-normalized) state, depending on the non-linearity of the map. In the latter case infinite ergodic theory plays a key role in the description of time averages. We will present physical applications of infinite ergodic theory in the context of stochastic Langevin dynamics [1] where the normalizing partition function diverges, and for a gas of laser cooled atoms [2]. This allows for the construction of thermodynamical relations in a non-equilibrium setting, provided that the dynamics is recurrent.

[1.] E. Aghion, D. A. Kessler, and E. Barkai Phys. Rev. Lett. 122, 010601 (2019).

[2.] E. Barkai, G. Radons, and T. Akimoto Transitions in the ergodicity of subrecoil-laser-cooled gases Phys. Rev. Lett. 127, 140605 (2021).

DY 4.3 Mon 10:15 H43

Towards a model-free inference of hidden states and transition pathways — ●XIZHU ZHAO¹, DMITRII E. MAKAROV², and ALJAZ GODEC¹ — ¹Max Planck Institute for Multidisciplinary Sciences, Göttingen, Germany — ²University of Texas at Austin, Austin, Texas, USA

Experiments on biophysical systems typically probe lower-dimensional observables, which are projections of high-dimensional dynamics. In order to infer a consistent model capturing the relevant dynamics of the system, it is important to detect and account for the memory in the dynamics. We develop a method to infer the presence of hidden states and transition pathways based on transition probabilities between observable states conditioned on history sequences for projected (i.e. observed) dynamics of Markov chains. The histograms conditioned on histories reveal information on the transition probabilities of hidden paths locally between any specific pair of states, including the duration of memory. The method can be used to test the local Markov property of observables. The information extracted is also helpful in inferring relevant hidden transitions which are not captured by a Markov-state model.

DY 4.4 Mon 10:30 H43

Interaction statistics in persistent Lotka-Volterra communities — ●JOSEPH BARON¹, THOMAS JUN JEWELL³, CHRISTOPHER RYDER⁴, and TOBIAS GALLA² — ¹University of Bath, UK — ²IFISC, Palma de Mallorca, Spain — ³University of Oxford, UK — ⁴University of Manchester, UK

One criticism that was levelled at Robert May's seminal ecological work, which posited random interactions to describe the stability of many-species ecological communities, was that such interactions may not arise naturally in any reasonable model of the ecosystem dynamics. In this talk, I discuss the kinds of interactions that arise between species in communities modelled by generalised Lotka-Volterra dynamics. Far from being iid random variables, there is an intricate structure of correlations between different species' interactions. These arise due to constraints on the species abundances that are imposed by the dynamics. I show that in order to correctly predict stability, one can no longer think of species as being statistically equivalent – a hierarchy amongst the species naturally emerges, even when the initial pool of species are statistically interchangeable. In a similar vein, I also show how the initial interaction network between species is warped by the Lotka-Volterra dynamics, changing the degree distribution, and introducing correlations between a species' connectivity and its interaction statistics. In the end, we see that the interactions in coexisting communities have non-trivial statistical interdependencies, and understanding this statistical structure can help us to understand which species are able to persist in a particular community.

DY 4.5 Mon 10:45 H43

Reduced order stochastic modeling of turbulent passive scalar mixing — ABHISHEK JOSHI, TOMMY STARICK, ●MARTEN KLEIN, and HEIKO SCHMIDT — BTU Cottbus-Senftenberg, Cottbus, Germany

Turbulent mixing is composed of chaotic stirring (macromixing) and molecular diffusion (micromixing). The detailed numerical modeling of turbulent mixing has remained a challenge since all relevant flow scales have to be represented in a computationally feasible manner. Map-based stochastic modeling approaches address this challenge by a radical abstraction, which is accomplished by dimensional reduction and utilization of generalized baker's maps to model turbulent fluid motions. Dimensional reduction introduces limitations, but the modeling approach offers interpretability of the emerging complexity and, hence, further physical insight into turbulent mixing. In this contribution, the Hierarchical Parcel Swapping (HiPS) [Kerstein, J. Stat. Phys. 153, 142–161 (2013)] and the One-Dimensional Turbulence (ODT) [Kerstein, J. Fluid Mech. 392, 277–334 (1999)] models are used to study turbulent mixing of passive scalars. Both models aim to represent the state space of 3-D turbulent mixing by a 1-D computational domain. HiPS is a fully event-based mixing model with prescribed sampling from a turbulent cascade, whereas ODT employs an energy-based rejection sampling only for macromixing such that a turbulent cascade is the result of a prescribed physical forcing mechanism. Capabilities of both models are demonstrated for canonical single and multiple passive scalar mixing cases using standalone model formulations across diffusivities.

DY 4.6 Mon 11:00 H43

Which methods are best suited for predicting chaotic time series? — ●ULRICH PARLITZ — Max Planck Institute for Dynamics and Self-Organization, Göttingen, Germany

Since the pioneering work in the 1980s on approximating the time evolution of dynamical systems using delay embedding, many methods for predicting univariate and multivariate chaotic time series have

been proposed and published. Recently, the prediction of chaotic time evolution has also been used to demonstrate the performance of novel machine learning algorithms, but in many cases only low-dimensional chaos is considered, as generated by the classical Lorenz-63 system. In this talk, different prediction methods will be contrasted and compared in terms of their prediction power and complexity when applied to low- and high-dimensional chaotic time series.

15 min. break

Invited Talk

DY 4.7 Mon 11:30 H43

Modelling the movements of organisms: Movement ecology meets active particles and anomalous diffusion — ●RAINER KLAGES — Centre for Complex Systems, School of Mathematical Sciences, Queen Mary University of London

Organisms living at very different spatio-temporal scales, from migrating in the microworld to foraging at the surface of the earth, typically all display random-looking movement patterns. Understanding these complex patterns by constructing mathematical models from data provides a fundamental challenge. In this talk I first review fundamental stochastic models for understanding movement data, like random walks, Langevin equations and active Brownian particles. On this basis experimental data for the movement paths of foraging sea turtles, migrating cells and bumblebee flights is analysed. For all three examples generalised overdamped Langevin equations are constructed from data revealing active and anomalous diffusive properties. I then put forward a generalised underdamped Langevin equation for modelling organismic movements, which blends key ingredients of the three fields of movement ecology, active particles and anomalous diffusion. I illustrate the application of this equation for constructing a stochastic model of bumblebee flights from experimental data and outline its theoretical foundation.

DY 4.8 Mon 12:00 H43

Self-diffusion anomalies of an odd tracer in soft-core media — PIETRO LUIGI MUZZEDDU¹, ●ERIK KALZ², ANDREA GAMBASSI^{3,4}, ABHINAV SHARMA^{5,6}, and RALF METZLER² — ¹University of Geneva — ²University of Potsdam — ³SISSA, Trieste — ⁴INFN, Trieste — ⁵University of Augsburg — ⁶IPF, Dresden

Odd-diffusive systems, characterised by broken time-reversal and/or parity symmetry, have recently displayed counterintuitive features such as interaction-enhanced dynamics in the dilute limit. Here we extend the investigation to the high-density limit of an odd tracer embedded in a soft Gaussian core medium (GCM) using a field-theoretic approach based on the Dean-Kawasaki equation. Our theory reveals that interactions can enhance the dynamics of an odd tracer even in dense systems. We demonstrate that oddness results in a complete reversal of the well-known self-diffusion (D_s) anomaly of the GCM. Ordinarily, D_s exhibits a non-monotonic trend with increasing density, approaching but remaining below the interaction-free diffusion, D_0 , ($D_s < D_0$) so that D_s approaches D_0 at high densities from below. In contrast, for an odd tracer, self-diffusion is enhanced ($D_s > D_0$) and the GCM anomaly is inverted, such that D_s approaches D_0 at high densities from above. The transition between the standard and reversed GCM anomaly is governed by the tracer's oddness, with a critical oddness value at which the tracer diffuses as a free particle ($D_s = D_0$) across all densities. — arXiv:2411.15552

DY 4.9 Mon 12:15 H43

Memory effects and non-linear responses in colloidal depinning — ARTHUR V. STRAUBE^{1,2} and ●FELIX HÖFLING^{2,1} — ¹Zuse-Institut Berlin — ²Institut für Mathematik, Freie Universität Berlin

Particle transport in inhomogeneous environments is complex and typically exhibits non-Markovian responses. The latter can be quantified by a memory function within the framework of the linear generalised Langevin equation (GLE). Here, we exemplify the implications of steady driving on the memory of a colloidal model system for Brownian motion in a corrugated potential landscape, a prototypical set-up to study depinning and non-linear responses far from equilibrium [1,2]. Relying on exact solutions of the model, we show that the random force entering the GLE displays a bias far from equilibrium, which corroborates a recent more general prediction. Based on Brownian dynamics simulations, we show that already moderate driving accelerates the decay of the memory function by several orders of magnitude in time. Moreover in equilibrium, the memory persists much longer than suggested by the timescales of the mean-square displacement. Furthermore, the memory function changes from a monotonic decay to non-monotonic, damped oscillations, which can be understood from a competition of confined motion and depinning. The simulated transport process also is pronouncedly non-Gaussian, which questions the usual Gaussian approximation of the random force in the GLE.

[1] A. V. Straube & F. Höfling, J. Phys. A **57**, 295003 (2024).

[2] A. V. Straube & F. Höfling, Phys. Rev. E **110**, L06260x (2024).

DY 4.10 Mon 12:30 H43

Non-Gaussian random forces in the generalized Langevin equation — ●HÉLÈNE A. COLINET and ROLAND R. NETZ — Freie Universität Berlin

The generalized Langevin equation (GLE), derived by projection from a general many-body Hamiltonian, exactly describes the dynamics of an arbitrary coarse-grained variable in a complex environment. The complementary force term (typically called a random force) describes the interaction with the environment, and is characterized by its second moment, linking it to the time-dependent friction memory kernel. For practical applications, this complementary force is commonly modeled as a Gaussian stochastic process, thus neglecting higher-order moments, which can become a bad approximation for non-linear systems. Leveraging advanced GLE extraction and simulation techniques, we explore the limitations of the Gaussian assumption and examine the role of non-Gaussian random forces in protein folding and conformational changes.

DY 4.11 Mon 12:45 H43

Chemo-mechanical motility modes of partially wetting liquid droplets — ●FLORIAN VOSS¹ and UWE THIELE^{1,2} — ¹Institute for Theoretical Physics, University of Münster, Germany — ²Center for Nonlinear Science, University of Münster, Germany

We consider a simple thermodynamically consistent model that captures the interplay between an autocatalytic reaction of chemical species on the free surface of a droplet, the solutal Marangoni effect and the physics of wetting in the presence of chemical fuel [1]. We find that a positive feedback loop between the local reactions and the Marangoni effect induces surface tension gradients, allowing for self-propelled droplets. Besides simple directional motion, we find crawling, shuttling and randomly moving droplets. We study the occurring dynamics and show how the observed states generically emerge from (global) bifurcations. We speculate that our results may also be relevant to the study of cell crawling [2] and self-propelled biomolecular condensates [3].

[1] Voss, F., Thiele, U., J. Eng. Math. 149, 2024

[2] Ziebert, F., Swaminathan, S., Aranson, I., J. R. Soc. Interface **9**, 2012

[3] Demarchi, L., Goychuk, A., Maryshev, I., Frey, E., Phys. Rev. Lett **130**, 2024