DY 7: Focus Session: Large Deviations and Rare Events I

The modeling and understanding of large deviations and rare events is of crucial importance in a wide range of real-world applications, including climate science, actuarial statistics (insurance statistics), natural disaster management, or the description of financial markets. At the same time, such effects are fundamental for the understanding of many systems in condensed-matter physics. In first-order phase transitions, for instance, the coexisting phases are connected by transition states involving droplet excitations whose probability is suppressed by dozens or hundreds of orders of magnitude as compared to the pure-phase peaks. Likewise, for many disordered systems the behavior of typical cases is incompatible with that of the average sample as the problem is described by very broad, heavy-tailed distributions, where averages are dominated by rare events. For many models, like the Kadar-Parisi-Zhang equation or random-graph properties, there has been considerable analytical progress in the last decade. Likewise, new or improved numerical techniques have been proposed that now allow for the treatment of previously inaccessible problems or regimes. This focus session is devoted to an update on the state of the art in this rapidly evolving area.

Organized by Alexander K. Hartmann (Oldenburg) and Martin Weigel (Chemnitz)

Time: Monday 9:30–12:45 Location: ZEU/0114

Invited Talk DY 7.1 Mon 9:30 ZEU/0114

Fast Rare Events in Exit Times Distributions of Jump Processes — •Raffaella Burioni — Department of Mathematics, Physics and Computer Sciences, University of Parma, Italy — INFN - Sezione di Milano Bicocca - Parma, Italy

Rare events in first-passage and exit-time statistics of jump processes can play a decisive role in triggering anomalous reactions and extreme responses in a wide range of systems. This is particularly relevant when jump lengths or waiting times follow broad, heavy-tailed distributions, for which rare events are not exponentially suppressed and can significantly affect macroscopic observables. Interestingly, in the presence of heavy-tailed distributions, large fluctuations follow the Big Jump Principle, a counterintuitive mechanism according to which rare events arise not from the accumulation of many small deviations, but from a single, dominant fluctuation.

We present a general framework for estimating the contribution of fast rare events to exit probabilities in jump processes with fat-tailed distributions. We apply this approach to discrete-time random walks, Lévy walks, and the Lévy-Lorentz gas, which are widely used to describe transport in biological systems and disordered media. We derive the scaling form of the probability distribution for fast exit events, in which the process leaves a finite interval over distances much larger than the typical scale and on timescales orders of magnitude shorter than the characteristic timescale of the dynamics. We discuss extensions to N independent walkers, where collective effects can further enhance the contribution of fast rare events to exit statistics.

DY 7.2 Mon 10:00 ZEU/0114

Rare Events, Many Searchers, and Fast Target Reaching in a Finite Domain — Elisabetta Ellettari[1], Giacomo Nasuti[1], •Alberto Bassanoni[1,2], Alessandro Vezzani[1,3], and Raffaella Burioni[1,2] — [1]University of Parma, Italy — [2]INFN, Parma associated group, Italy — [3]IMEM-CNR, Parma, Italy

Finding a target in a complex environment is a fundamental challenge in nature. An effective strategy to reduce the time needed to reach a target is to deploy many searchers, increasing the likelihood that at least one will succeed by using the statistics of rare events. When the underlying stochastic process involves broadly distributed step sizes, rare long jumps dominate the dynamics, making the use of multiple searchers particularly powerful. We investigate the statistics of extreme events for the mean first passage time in a system of N independent walkers moving with jumps distributed according to a power law, where target-reaching is governed by single, large fluctuations. We show that the mean first passage time of the fastest walker scales as $\langle \tau_N \rangle \sim 1/N$, representing a dramatic speed-up compared to classical Brownian search strategies. We derive a scaling law relating the number of walkers required to reach a target within a given time to the size X of the search region. As an application, we model biological fertilization, predicting how the optimal number of spermatozoa scales with uterus size across species. Our predictions match empirical data, and this theory applies broadly to any population of searchers operating within a region of size X, providing a universal framework for efficient search in disordered environments.

DY 7.3 Mon 10:15 ZEU/0114

Large deviations and uncertainty relations for self-interacting jump processes — •Francesco Coghi[1,2], Amarjit Budhiraja[3], and Juan P. Garrahan[1,2] — [1]School of Physics and Astronomy, University of Nottingham, Nottingham, NG7 2RD, UK — [2]Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, University of Nottingham, Nottingham, NG7 2RD, UK — [3]Department of Statistics and Operations Research, University of North Carolina, Chapel Hill, NC 27599, USA

Self-interacting jump processes are stochastic systems where transition rates depend on the process' own empirical occupation measure – that is, the time-averaged distribution of the states visited by the process – introducing a feedback that breaks Markovianity. In this talk, I will present a large deviation framework for such systems. Specifically, I will derive the level-2.5 rate functional describing the joint fluctuations of occupation and flux, obtained via an exponential tilting construction extended to the non-Markovian setting. From this, I will show how memory modifies fluctuation bounds, leading to kinetic and thermodynamic uncertainty relations that generalise those of standard Markov processes. I will conclude with two examples that illustrate how feedback reshapes fluctuation-dissipation trade-offs: a minimal two-state model and a collective exploration process inspired by ant dynamics.

DY 7.4 Mon 10:30 ZEU/0114

A pedestrian's approach to large deviations in semi-Markov processes with an application to entropy production — •Jonas H. Fritz, Alexander M. Maier, and Udo Seifert — II. Institut für Theoretische Physik, Universität Stuttgart, 70550 Stuttgart, Germany

Semi-Markov processes play an important role in the effective description of partially accessible systems. They occur, for instance, in coarse-graining procedures such as state lumping and when analyzing waiting times between few visible Markovian events. The finite-time measurement of any coarse-grained observable in a stochastic system depends on the specific realization of the underlying trajectory. Moreover, the fluctuations of such observables are encoded in their rate function that follows from the rate function of the empirical measure and the empirical flow in the respective process. Derivations of the rate function of empirical measure and empirical flow in semi-Markov processes with direction-time independence (DTI) exist in the mathematical literature, but have not received much attention in the stochastic thermodynamics community. We present an accessible derivation of the rate function of the tuple frequency in discrete-time Markov chains and extend this to the rate function of the empirical semi-Markov kernel in semi-Markov processes without DTI [1]. From this, we derive an upper bound on the rate function of the empirical entropy production rate, which leads to a lower bound on the variance of the mean entropy production rate measured along a finite-time trajectory. We illustrate these analytical bounds with simulated data. [1] Alexander M. Maier, Jonas H. Fritz and Udo Seifert, arXiv:2509.15077, 2025

DY 7.5 Mon 10:45 ZEU/0114

Concentration of observables in slow-fast dynamical systems with noise — •Xizhu Zhao[1,2] and Aljaž Godec[1,2] —

[1]Mathematical Physics and Stochastic Dynamics, Institute of Physics, University of Freiburg, Freiburg, Germany — [2]Max Planck Institute for Multidisciplinary Sciences, Göttingen, Germany

Stochastic differential equations involving separated timescales play an important role in modeling the dynamics of complex systems in physics, biology, and climate science. We explore the concentration of observables, i.e. functions or functionals of dynamical variables, in slow-fast dynamical systems with noise. Using stochastic analysis and singular perturbation theory, we determine the domains of concentration for observables in systems with a stable manifold or undergoing a bifurcation. We also derive bounds for the distribution of the first-exit time from these domains. The results show that the concentration of observables exhibits behavior distinct from that of microscopic sample paths.

15 min. break

Invited Talk DY 7.6 Mon 11:15 ZEU/0114

Large deviations in resetting Brownian motions — •Satya Majumdar — CNRS, LPTMS, Universite Paris-Saclay, Orsay 91405, France

After a brief introduction to large deviations, I will focus on one dimensional resetting Brownian motions, first on a single particle and then on a gas of multiparticles. I will show that for a single partcle, while the position distribution of a resetting Brownian motion approaches a stationary form at long times, the relaxation to this stationary state is unusual: it is described by a large deviation function that undergoes a second order dynamical phase transition. The same rate function also shows up in the distribution of the maximum up to time t of a set of independent resetting Brownian motions with a distributed initial positions. I will discuss the difference between the 'quenched' and 'annealed' initial positions and how they affect the statistics of the maximum.

DY 7.7 Mon 11:45 ZEU/0114

Large deviations and condensation in the cost of stochastic resetting — •John C. Sunil[1], Martin R. Evans[1], Richard A. Blythe[1], and Satya N. Majumdar[2] — [1]SUPA, School of Physics and Astronomy, University of Edinburgh, Peter Guthrie Tait Road, Edinburgh EH9 3FD, United Kingdom — [2]Université Paris-Saclay, CNRS, LPTMS, 91405 Orsay, France

Searching for misplaced objects is a task that we are all familiar with. In particular, the way we search for a misplaced object, such as keys, is by repeatedly resetting to the object*s last known location. It has been shown that such resetting expedites the mean time to complete the search process. However, in practice, resets must also incur costs, whether in time, energy, or money. This motivates examining the probabilities of rare cost fluctuations that deviate significantly from typical behaviour. In this talk, I will demonstrate the effect of costly resets and the large deviations exhibited by the tails of the total cost distribution using an exactly solvable model of diffusion with stochastic resetting. Further, I will discuss the eventual breakdown of the large deviation principle for certain classes of cost, resulting in a phase transition known as condensation.

References

(i) J. C. Sunil, R. A. Blythe, M. R. Evans and S. N. Majumdar, The cost of stochastic resetting, J. Phys. A: Math. Theor. 56 (2023) 395001.

(ii) M. R. Evans and J. C. Sunil, Stochastic Resetting and Large Deviations, SciPost Phys. Lect. Notes, vol. 103, 2025.

DY 7.8 Mon 12:00 ZEU/0114

Perturbative framework for finding the transition rates for active particles — •Vito Seinen[1], Peter Bolhuis[1], Daan Crommelin[2], Michel Mandjes[3], and Sara Jabbari-Farouji[1] — [1]University of Amsterdam — [2]Centrum Wiskunde & Informatica — [3]University Leiden

Activated escape or transition rates between metastable states have been a growing topic of research due to their relevance in many physical processes and their roles in certain non-equilibrium phase transitions. In the context of motility-induced phase separation (MIPS), such active transition rates are known to govern the nucleation processes that initiate phase separation. To gain insight into these phenomena, we develop a projection-operator formalism for a generic model of an active particle in a multi-well potential. We compute the transition rates perturbatively in two asymptotic regimes: one in which the dynamics of the activity (the driving process) is much faster than that of the particle (the driven process), and one in which the activity dynamics is much slower. From these asymptotic expansions, we construct an approximation that interpolates across the intermediate regime, thereby capturing all parameter ranges relevant in the rare-event limit. Our results provide deeper understanding of the complex dependence of active transition rates on the magnitude of the activity, the thermal noise strength, and the dynamical timescale of the active (driving) process*its persistence time

DY 7.9 Mon 12:15 ZEU/0114

Large deviation properties of Brownian particles in switching harmonic traps — •Chinmay Pradeep Chandratre and Alexander K. Hartmann — Institut of Physics, University of Oldenburg, Oldenburg (Germany)

For N independent particles in a harmonic trap with stiffness switching between μ_1 and μ_2, a complete characterization of the non-equilibrium steady state including the computation of joint distribution of particle positions has been performed under the stochastic-resetting protocol [1]. In the large-N limits, this allows for the computation of observables such as the distribution of the position M_k of the k-th rightmost particle (Extreme value statistics), spacing distributions $D_k = M_k - M_{k+1}$ and the particle count $N_L = |\{\, i : |x_i| \leq L \,\}|$ for an interval $[-L, L]$. However, the finite-N regime with significant large-deviation corrections is analytically unknown. The numerical limitations in resolving the support of the distribution, especially in the tails, are circumvented through specialized large-deviation algorithms [2] by deploying modified Markov Chain Monte Carlo methods. The distributions $P(M_k)$, $P(D_k)$ and $P(N_L)$ and corresponding rate functions were obtained numerically from simulations performed for several system sizes. Conditional distribution rendered further insight into the correlations between M_k, D_k, and N_L.

[1] Biroli, Marco and Kulkarni, Manas and Majumdar, Satya N. and Schehr, Grégory, Phys. Rev. E **109**, 032106 (2024)

[2] A.K. Hartmann, Phys. Rev. E **89**, 052103 (2014)

DY 7.10 Mon 12:30 ZEU/0114

Optimal escape processes of run-and-tumble particles — Karthik Cheruvary[2], •Rafael Diaz Hernandez Rojas[1], and Peter Sollich[1] — [1]University of Göttingen — [2]IISER Pune

Run-and-tumble (RT) motion is a fundamental mode of self-propulsion observed across diverse biological systems from bacterial chemotaxis to immune cell migration. Modelling this non-equilibrium system is challenging due to the non-Gaussian noise governing the self-propulsion direction, θ. To address this, we investigate the trajectories of a particle confined by a potential in $d = 2$ and subject to both thermal fluctuations and RT motion. We incorporate tumbling by modelling changes in θ as Poisson shot noise process with rate λ and generic distribution of angle changes, Δ. Employing a path-integral formalism in the weak noise limit, we derive exact equations of motion for all degrees of freedom, valid for arbitrary λ and distributions of Δ. This allows us to map the problem of finding the most likely escape trajectory to the minimisation of an appropriate action. Our analysis reveals that the effects of the non-Gaussian noise are more prominent in the limit of small λ, which corresponds to highly persistent RT motion. There, we show analytically that, regardless of the distribution of Δ, RT escape paths are exponentially more probable –exhibiting a lower action– than their active Brownian particle equivalent. The optimal RT trajectories are non-trivial and can exhibit discontinuities or very rapid changes in θ, implying that RT particles will exploit their tumbling ability to optimize escape from potential wells. We verify our theoretical predictions through extensive numerical simulations.