

## DY 8: Nonlinear Dynamics, Synchronization, and Chaos

Time: Monday 9:30–12:30

Location: ZEU/0118

DY 8.1 Mon 9:30 ZEU/0118

**Coherence properties of collective modes in oscillator networks** — ●ARKADY PIKOVSKY — University of Potsdam, Potsdam, Germany

Synchronization transition in populations of coupled oscillators is manifested by an appearance of a global oscillating mode. While oscillations are perfect in the thermodynamic limit, for finite ensembles the collective mode is subject to fluctuations. Here we focus on the properties of diffusion of the global phase, which determine coherence time of the collective oscillations. We discuss scaling of the diffusion constant with the system size, and describe cases where the coherence is perfect.

DY 8.2 Mon 9:45 ZEU/0118

**Phase locking and multistability in the topological Kuramoto model on cell complexes** — ●IVA BACIĆ<sup>1,2</sup>, MICHAEL T. SCHAUB<sup>3</sup>, JÜRGEN KURTHS<sup>4</sup>, and DIRK WITTHAUT<sup>1,5</sup> — <sup>1</sup>Institute of Climate and Energy Systems: Energy Systems Engineering (ICE-1), Forschungszentrum Jülich, 52428 Jülich, Germany — <sup>2</sup>Institute of Physics Belgrade, University of Belgrade, Serbia — <sup>3</sup>RWTH Aachen University, Aachen, Germany — <sup>4</sup>Potsdam Institute for Climate Impact Research (PIK), Potsdam, Germany — <sup>5</sup>Institute for Theoretical Physics, University of Cologne, 50937 Köln, Germany

Higher-order group interactions fundamentally shape the dynamics and stability of oscillator networks. The topological Kuramoto model captures these effects by extending classical synchronization models to include interactions between cells of arbitrary dimension within simplicial and cell complexes. We present the topological nonlinear Kirchhoff conditions algorithm, a nonlinear generalization of Kirchhoff's circuit laws, that systematically identifies all phase-locked states in the topological Kuramoto model and reveals how higher-order topology governs multistability. Applying this framework to rings, Platonic solids, and simplexes, we uncover structural cascades of multistability inherited across dimensions, and demonstrate that cell complexes can exhibit richer multistability patterns than simplicial complexes of equal dimension. We find evidence hinting at universal multistability classes. Our results reveal how higher-order interactions affect synchronization and open new directions for understanding collective dynamics in systems with non-pairwise interactions.

DY 8.3 Mon 10:00 ZEU/0118

**From quaternion order parameter to extreme synchronization transitions** — ●MORITZ THÜMLER<sup>1</sup>, SEUNGJAE LEE<sup>1</sup>, LENNART J. KUKLINSKI<sup>1</sup>, and MARC TIMME<sup>1,2</sup> — <sup>1</sup>Chair of Network Dynamics, Center for Advancing Electronics Dresden (cfaed) and Institute of Theoretical Physics, TUD Dresden University of Technology, 01062 Dresden, Germany — <sup>2</sup>Lakeside Labs, Lakeside B04b, 9020 Klagenfurt, Austria

Synchrony plays a fundamental role in the operation of many systems across physics, biology, and engineering. Fifty years after the introduction of the paradigmatic Kuramoto model\*which provides a mathematically tractable framework for studying transitions to synchrony\*several open questions remain, particularly concerning finite-size systems. An analytic continuation of the Kuramoto model from real to complex variables has been proposed to address this challenge [1]. Here, we introduce a generalized quaternion order parameter that captures complex-valued state vectors and demonstrate its effectiveness by uncovering a link between real-valued Kuramoto dynamics and complex locked states calculated via self-consistency and newly uncovered explosive transitions to synchrony [2].

[1]\*Moritz Thümmler et al., Synchrony for Weak Coupling in the Complexified Kuramoto Model, Physical Review Letters 130:187201 (2023).

[2] Lee, Seungjae, Kuklinski, Lennart J., Thümmler, Moritz and Timme, Marc. "Hopf-induced desynchronization" Zeitschrift für Naturforschung A, vol. 80, no. 11, 2025,

DY 8.4 Mon 10:15 ZEU/0118

**Transition to turbulence in wind turbine wakes: synchronisation and non-linear dynamics** — ●THOMAS MESSMER, MICHAEL HÖLLING, and JOACHIM PEINKE — Carl von Ossietzky Universität Oldenburg, School of Mathematics and Science, Institute of Physics, Oldenburg, Germany

The wake of a wind turbine is a typical shear flow that transitions from near- to far-wake, ultimately reaching fully developed turbulence. In this study, we investigate experimentally in a wind tunnel the wake dynamics of a periodically excited model wind turbine\*a scenario motivated by applications such as floating wind turbines and periodic control strategies. Our findings reveal distinct excitation regimes with unique wake behaviours.

In one regime, the wake synchronises with the excitation period, forming an organised flow structure that accelerates the transition to turbulence. In another regime, nonlinear quasi-periodic dynamics emerge, characterised by the formation of multiple coherent structures that interact nonlinearly and also accelerate the transition to the far wake.

At the conference, we will discuss the mechanisms underlying synchronisation and quasi-periodic dynamics, as well as their implications for the fundamental problem of the transition to turbulence.

## Invited Talk

DY 8.5 Mon 10:30 ZEU/0118

**Mean-field approach to finite-size fluctuations in coupled oscillator systems** — ●OLEH OMELCHENKO<sup>1</sup> and GEORG GOTTWALD<sup>2</sup> — <sup>1</sup>University of Potsdam, Germany — <sup>2</sup>University of Sydney, Australia

Networks of coupled phase oscillators are one of the most studied dynamical systems with numerous applications in physics, chemistry, biology, and engineering. A variety of methods exists to explain their properties and dynamics in the thermodynamic limit, when the network size tends to infinity. However, the behavior of such systems in the more realistic case of a finite number of oscillators still remains poorly understood. In this talk, we revisit the paradigmatic Kuramoto-Sakaguchi model describing synchronization transitions in networks of all-to-all coupled heterogeneous phase oscillators, and propose an ab initio approach for characterizing analytically the statistical properties of finite-size fluctuations in this system. Our framework is applicable to any stationary partially synchronized state and does not require any prior knowledge about its structure. Moreover, it is sufficiently general such that it can be applied to a broader class of interacting particle systems.

## 15 min. break

DY 8.6 Mon 11:15 ZEU/0118

**Energy Transfer in a Coupled Duffing System** — ●MARTYNA SEDLMAYR and ANDRZEJ RYSAK — Faculty of Mechanical Engineering, Lublin University of Technology, 36 Nadbystrzycka St., 20-618 Lublin, Poland

The Duffing system is a non-linear prototypical chaotic system, of interest in studies of damping and energy harvesting. In this work we analyse modifications to the energy transfer in the system when it is magnetically coupled to a harmonic oscillator. The form of the magnetic interaction is obtained from a fit to experimental data. Numerical analyses then determine the power either dissipated by, or supplied to, the Duffing system by its individual components. In all simulations, the Duffing potential itself is not changed. Our analysis focuses on assessing the impact of the coupled oscillator on the energy efficiency of the system. We demonstrate the effects of changing the value of the mass, the damping and the elasticity of the perturbing system. We focus on searching for configurations of the perturbation for which an increase in efficiency occurs, which can be caused by either modifying the dynamics or by providing additional energy.

DY 8.7 Mon 11:30 ZEU/0118

**Transient Signatures of Flow-Topological Transitions in Non-linear Quantum Oscillators** — ●ALEJANDRO S. GÓMEZ and JAVIER DEL PINO — Department of Theoretical Condensed Matter Physics, (IFIMAC), Universidad Autónoma de Madrid

Non-equilibrium phases in driven-dissipative systems are ubiquitous. Often, they appear as classical fixed point attractors and limit cycles, with standard phase transitions triggered by local instabilities. Yet the flow-topology framework [1] shows that transitions can also arise from nonlocal reorganizations, leaving attractors unchanged and emerging only in transients. Stationary quantum states still reflect these structures as probability hotspots, and local changes can induce Liouvillian

spectral degeneracies. However, how the nonlocal, transient reorganizations manifest in the Liouvillian remains open.

In this talk, I will introduce a topological framework that extends the classification of fixed points [1] to capture the full phase-space connectivity considering also limit cycles. By linking open quantum dynamics to the classification of Morse-Smale flows [2], Crucially, we demonstrate that our graph invariant exposes transitions the Liouvillian spectrum misses altogether, related to global flow-topology transitions. I illustrate this framework by mapping the dissipative phases of a two-photon-driven Kerr resonator with added gain.

[1] G. Villa et al., Topological classification of driven-dissipative nonlinear systems, *Sci. Adv.* (2025).

[2] A. A. Oshemkov, Classification of Morse-Smale flows on two-dimensional manifolds, *Sbornik Math.* (1998).

DY 8.8 Mon 11:45 ZEU/0118

**The disordered logistic map** — JOSEPH W. BARON<sup>1</sup> and TOBIAS GALLA<sup>2</sup> — <sup>1</sup>Department of Mathematical Sciences, University of Bath, Bath, BA2 7AY, UK — <sup>2</sup>Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB), 07122 Palma de Mallorca, Spain

The logistic map is a central model of low dimensional chaos. In disordered systems with many degrees of freedom on the other hand, one finds high-dimensional chaos, induced by heterogeneous interactions.

Here, we study a system of many logistic maps with quenched random interactions. Using dynamic mean-field theory, random matrix theory and simulations we show that minimal disorder removes the period-doubling cascade in the conventional logistic map. The cascade is replaced by a sudden onset of chaos. We find a separate and qualitatively different transition to chaos with more smoothly evolving trajectories. The power spectra of fluctuations exhibit different power-law behaviour near the two transitions. In a disordered Henon map we find very similar behaviour.

Our results show that even minimal disorder can severely disrupt the behaviour of well-known dynamical systems. The work also shows that discrete-time disordered systems can exhibit rather different behaviour from their continuous-time cousins [1]. Disordered maps highlight that different chaotic behaviours can be observed in one single system, and that each of these behaviours leaves a fingerprint before chaos sets in.

[1] J. W. Baron, T. J. Jewell, C. Ryder, and T. Galla, Breakdown of random-matrix universality in persistent Lotka-Volterra communities, *Physical Review Letters* 130, 137401 (2023).

DY 8.9 Mon 12:00 ZEU/0118

**Transient nature of recurrence based local dimension estimates from finite time series** — REIK V. DONNER — Magdeburg-Stendal University of Applied Sciences, Magdeburg, Germany — Pots-

dam Institute for Climate Impact Research, Potsdam, Germany

Local fractal dimensions are a widely used concept in dynamical system theory. However, comparing their empirical spatial patterns across published works reveals surprising inconsistency with theoretical expectations. This points to the fact that corresponding estimates obtained from finite time series are notoriously spurious. As an example, a very long trajectory of the Lorenz-63 system in its chaotic regime is subsampled in two different ways: generating ensembles of individual state vectors drawn independently at random from the complete record versus ensembles of contiguous time series segments. In the latter situation, which is commonly present in time series analysis, one obtains a broad distribution and non-random spatial pattern of local dimension estimates, while consideration of independent samples provides much more narrowly distributed and spatially unstructured estimates. While finite-time estimates may still be useful for identifying transient structures caused by the proximity to unstable periodic orbits or similar dynamically invariant objects, their practical interpretation for real-world time series should be carefully reconciled.

DY 8.10 Mon 12:15 ZEU/0118

**Two-dimensional turbulent condensates without bottom drag**

— ADRIAN VAN KAN<sup>1</sup>, ALEXANDROS ALEXAKIS<sup>2</sup>, and EDGAR KNOBLOCH<sup>3</sup> — <sup>1</sup>Department of Mathematics, Texas A&M University, College Station, USA — <sup>2</sup>Laboratoire de Physique de l'Ecole Normale Supérieure, ENS, Université PSL, CNRS, Paris, France — <sup>3</sup>Department of Physics, UC Berkeley, Berkeley, California, USA

The extent to which statistical equilibrium theory applies to driven dissipative dynamics remains an important open question in many systems. We use extensive direct numerical simulations of the incompressible two-dimensional (2D) Navier-Stokes equation to examine the steady state of large-scale condensates in 2D turbulence at finite Reynolds number  $Re$  in the absence of bottom drag. Large-scale condensates appear above a critical Reynolds number  $Re_c \approx 4.19$ . For  $Re \gtrsim Re_c$ , we find a power-law scaling of the energy with  $Re - Re_c$ , with the energy spectrum at large scales following the absolute equilibrium form proposed by Kraichnan. At larger  $Re$ , the energy spectrum deviates from this form, displaying a steep power-law range at low wave numbers with exponent  $-5$ , with most of the energy dissipation occurring within the condensate at large scales. We show that this spectral exponent is consistent with the logarithmic radial vorticity profile of the viscously saturated condensate predicted by quasilinear theory. Our findings shed new light on the classical problem of large-scale turbulent condensation in forced dissipative 2D flows in finite domains, showing that the large scales are close to equilibrium dynamics in weakly turbulent flows but not for strong condensates ( $Re \gg 1$ ).